Far-field sheaths due to fast waves incident on material boundaries*

D. A. D'Ippolito and J. R. Myra

Lodestar Research Corporation, 2400 Central Avenue, Boulder, Colorado 80301

E. F. Jaeger and L. A. Berry

Oak Ridge National Lab, Oak Ridge, Tennessee

May, 2007

Presented at the *17th Topical Conference on Radio Frequency Power in Plasmas,*
May 7-9, 2007, Clearwater, Florida

DOE/ER/54392-39 & ER/54823-3 LRC-07-114

Far Field Sheaths Due to Fast Waves Incident on Material Boundaries

D. A. D'Ippolito,[1] J. R. Myra,[1] E. F. Jaeger,[2] and L. A. Berry,[2]

[1]Lodestar Research Corporation, Boulder, Colorado; [2]ORNL, Oak Ridge, Tennessee

Abstract. The problem of far-field sheath formation is studied with a new quantitative 1D model. These sheaths occur when unabsorbed fast waves (FW) are incident on a conducting surface not aligned with a flux surface. Use of a nonlinear sheath BC gives self-consistent solutions for the wave fields and sheath, and incorporates a sheath plasma wave (SPW) resonance which enhances the sheath potential. The model is applied to edge fields computed by the AORSA-1D full-wave code for a typical D(H) minority heating scenario. This work indicates the conditions under which far-field sheaths can explain some of the "missing power" (low heating efficiency) and rf-specific impurity generation in ICRF experiments.

Keywords: ICRF, sheath BC, sheath plasma waves, far field sheaths, heating efficiency
PACS: 52.40.Kh, 52.40.Fd, 52.40.Hf, 52.50.Qt, 52.35.Mw

INTRODUCTION

It is generally acknowledged that rf sheath effects are important limiting factors in ICRF heating and current drive, especially in low-$k_{||}$ phasing. A great deal of experimental and theoretical work has been devoted to studying sheath effects [1,2] but a *quantitative* method for calculating the rf sheath potential and its nonlinear consequences is still lacking. This paper is the first step in developing a quantitative description of *"far-field" sheaths* [3], which form on surfaces far from the antenna when the single pass absorption is low and the unabsorbed fast waves (FW) encounter material surfaces not coincident with flux surfaces. Important questions are: Do far-field sheaths contribute significantly to the "missing power" (reduced core heating efficiency) in ICRF experiments? Do they contribute to the observed phasing dependence of the heating efficiency? The single pass damping (and isolation of the far wall from the FW fields) increases with $k_{||}$, so one would expect the "missing power" due to far-field sheaths to decrease with $k_{||}$ (as observed in many experiments). This line of thinking motivates the development of the model described here.

WAVE SCATTERING MODEL

Consider the problem of a propagating fast wave in 1D geometry encountering either (i) a conducting wall or (ii) a sheath, modeled as a thin, lossy vacuum region [4]. We assume that an incident FW travels in the negative x-direction and couples to additional rf waves upon encountering the boundary at x = 0. The boundary is *not* assumed to coincide with a magnetic flux surface, i.e.

$$\mathbf{s}\cdot\mathbf{b} \neq 0 \quad , \tag{1}$$

where $\mathbf{s} = \hat{e}_x$ is the unit vector normal to the sheath (pointing into the plasma), and **b** = **B**/B is the unit vector along the B-field direction. Thus, the B field mismatch with the wall is proportional to $\mathbf{s} \cdot \mathbf{b} = B_x / B$. We assume that the density n(x) is constant near the boundary, permitting an analytic treatment of the wave coupling.

In general, the incident FW with amplitude E_0 cannot satisfy the BC at the wall or sheath without coupling to a reflected FW E_1 and a slow wave (SW) E_2 at the boundary. Thus, the total rf electric field is given by

$$\mathbf{E} = e^{i(k_y y + k_z z - \omega t)} \sum_{j=0}^{2} E_j \mathbf{e}_j \, e^{i k_{xj} x} \tag{2}$$

where the $\mathbf{e}_j$ (without carat superscripts) are the unit wave polarization vectors. The tangential component $\mathbf{k}_t$ = (k_y, k_z) of the wavevector is specified and the k_x components are obtained from the appropriate dispersion relations. The rf **E** field is subject to a sheath BC [4], treating the sheath as a thin vacuum layer,

$$\mathbf{E}_t = \nabla_t (\Delta \, D_n) \quad , \tag{3}$$

which contains the metal wall BC as a special case ($\Delta \to 0$). Here, Δ is the time-averaged sheath width, and the term on the RHS is related to the sheath capacitance. Combining Eqs. (2) and (3) yields the solution

$$\text{FW}: \quad E_1 = E_0 \, \frac{\mathbf{s} \cdot \mathbf{g}_2 \times \mathbf{g}_0}{\mathbf{s} \cdot \mathbf{g}_1 \times \mathbf{g}_2} \quad , \quad \text{SW}: \quad E_2 = E_0 \, \frac{\mathbf{s} \cdot \mathbf{g}_0 \times \mathbf{g}_1}{\mathbf{s} \cdot \mathbf{g}_1 \times \mathbf{g}_2} \quad , \tag{4}$$

$$\mathbf{g}_j = \mathbf{e}_j - i \Delta \left(\mathbf{s} \cdot \boldsymbol{\varepsilon}_j \cdot \mathbf{e}_j \right) \mathbf{k}_j \quad . \tag{5}$$

where the fields, the Stix plasma dielector tensor $\boldsymbol{\varepsilon}_j$, and the wave polarizations $\mathbf{e}_j$ are all defined on the plasma side of the sheath-plasma interface. Note that even in the absence of the sheath term, satisfying the metal wall BC requires E_1 and E_2; the presence of the sheath term leads to an additional effect, called the "sheath plasma wave resonance" [5, 6], discussed below.

For an assumed sheath width Δ, the sheath potential is obtained by integrating E_x across the sheath layer to obtain

$$\Phi_{rf} \equiv -\Delta \left| \sum E_j \, \mathbf{s} \cdot \boldsymbol{\varepsilon}_j \cdot \mathbf{e}_j \right| \quad . \tag{6}$$

The self-consistent values of Φ_{rf} and Δ are obtained by imposing the Child-Langmuir Law

$$\Delta = \lambda_D \left(e\Phi_{sh} / T_e \right)^{3/4} \quad , \tag{7}$$

as an additional constraint, where $\Phi_{sh} = \Phi_{rf} + \Phi_B$ (Bohm sheath potential). Nonlinear rootfinding is used to find the value of Φ_{rf} that satisfies Eqs. (6) and (7) for the given Δ. All of the numerical results shown here are self-consistent in this sense.

SHEATH PLASMA WAVES

When the sheath width Δ is large enough that the two terms on the RHS in Eq. (5) are comparable in $\mathbf{g}_j$, the denominator $\mathbf{s} \cdot \mathbf{g}_1 \times \mathbf{g}_2$ can become small in Eq. (4). This implies the existence of a resonance, which produces a large sheath potential for some locus of points in the $\mathbf{n}_\perp = (n_y, n_z)$ parameter space. (The SPW resonance requires $\Delta(\mathbf{s}\cdot\boldsymbol{\varepsilon}_j\cdot\mathbf{e}_j)\mathbf{k}_j \sim 1$, which for typical parameters means $|\mathbf{n}_\perp| >> 1$, where the index of refraction is defined as $\mathbf{n} = \mathbf{k}c/\omega$.) Physically, the "sheath-plasma-wave" (SPW) resonance occurs because the inductive plasma current into the sheath and the capacitive current across the sheath form an LC circuit, with the possibility of an LC resonance for some parameters [5]. These surface waves were studied and simulated in [6] in studying near-field sheaths on IBW antennas, and the present work suggests that they also occur in far-field sheaths arising from low-single-pass FW heating.

The resonant enhancement of the self-consistent solutions for the SW field E_2 and the sheath potential Φ_{rf} has been confirmed by a survey of parameter space. In the limit $n_y >> n_z$, $|\mathbf{s}\cdot\mathbf{g}_1\times\mathbf{g}_2|$ can be small because $\mathbf{g}_1\times\mathbf{g}_2$ can be orthogonal to $\mathbf{s}$, although neither $\mathbf{g}_1$ or $\mathbf{g}_2$ is individually small, so in this limit the SPW involves a coupling of the FW and the SW and is therefore fundamentally *electromagnetic*. This limit can occur when a bumpy wall generates high k_y components (larger than those launched by the antenna) by the process of linear mode coupling. In the limit $n_x >> n_z >> n_y$, the SPW is an *electrostatic* slow wave satisfying $n_\perp \sim n_x >> n_\|$, and the plasma approaches resonance because the tangential component of $\mathbf{g}_2$ is small.

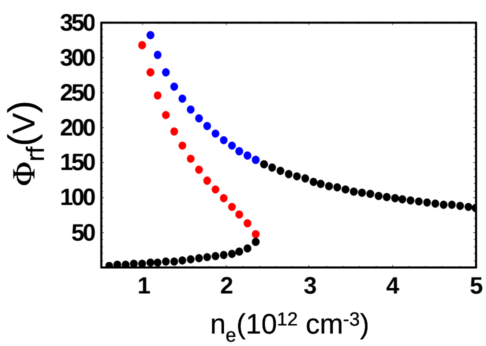

FIGURE 1. Sheath potential Φ_{rf} as a function of the plasma density n_e just outside the sheath for $n_y = 30$, $n_z = 6$ and $B_x/B = 0.2$. The existence of multiple roots is due to the physics of the sheath-plasma-wave (SPW) resonance, as discussed in the text. The hysterisis curve suggests an extreme sensitivity to the density at the wall.

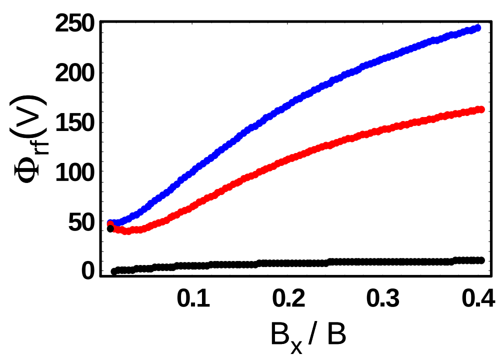

FIGURE 2. Sheath potential Φ_{rf} as a function of the magnetic field mismatch B_x/B. The upper two roots are associated with the SPW resonance. The parameters are $n_y = 20$, $n_z = 10$, $n_e = 2 \times 10^{12}$ cm^{-3}.

The roots for Φ_{rf} are obtained by solving the nonlinear relation obtained from Eqs. (6) and (7). In Fig. 1, we plot Φ_{rf} as a function of the density n_e at the wall. The other parameters are T_e = 10 eV, B = 30 kG, f = 60 Mhz, n_y = 30, n_z = 6, B_x/B = 0.2, and E_{y0} = 100 V/cm. Note that: (1) there are 3 roots for a restricted range of density at the wall; (2) there is an abrupt change in the value of the highest root at a critical density n_c (typical hysterisis pattern); (3) for $n > n_c$ the highest root has a large sheath potential ($e\Phi_{rf}/T_e \sim \Phi_{rf}/\Phi_B >> 1$) and would cause substantial power dissipation by ions accelerated down the sheath potential and into the wall [7]. The sensitivity of Φ_{rf} to the density suggests that the "missing power" lost through this mechanism would be sensitive to the conditioning of the machine and the recycling physics.

APPLICATION AND DISCUSSION

The wave scattering (WS) model was applied to a low-single-pass DIII-D scenario: D(H) minority ion heating with 2% H, f = 60 MHz, B = 32.5 kG (at HFS wall), $n_e = 2 \times 10^{12}$ cm^{-3}, toroidal mode number = 13, $k_y = 3$ m^{-1} ($\Rightarrow n_y = 2.4$ and $n_z = 6.3$), and $P_{rf} = 1$ MW. The AORSA-1D code [8] was used to calculate the unabsorbed FW E_{y0} and wavevector k_{x0} at the far wall. For 1 MW of launched power, the FW amplitude was approximately 50 V/cm at the wall. The sheath BC was taken into account by using the wave scattering model to calculate the self-consistent values Δ and Φ_{rf} for the specified incident FW amplitude. In the WS model, larger indices were used than in the wave propagation code, viz. $n_y = 20$ and $n_z = 10$, to account for linear mode coupling due to a bumpy wall (giving large values in the local k spectrum). The corresponding scale lengths are $L_y = 25$ cm and $L_z = 50$ cm, typical of a mismatch between the shape of the flux surface and that of a localized bump in the wall.

In Fig. 2, we plot Φ_{rf} as a function of the mismatch, B_X/B, to study the sensitivity to the magnetic geometry. Again there are multiple roots, and the higher roots are due to the SPW resonance. The sheath power losses to plasma-facing components in contact with rf fields increase by the factor Φ_{rf} / Φ_B. The SPW roots in Fig. 2 have large values of Φ_{rf} and could give rise to localized regions of power dissipation and hot spots. Even the lowest root yields a significant sheath potential, $\Phi_{rf} \sim 10$ eV $\sim \Phi_B$, and could contribute to edge power dissipation.

A 2D treatment is needed to accurately assess the global sheath power dissipation. The present work shows that the SPW resonance can produce locally large far-field sheath potentials. Studying this physics quantitatively in 2D and 3D rf codes is one of the goals of the rf SciDAC project.

ACKNOWLEDGMENTS

This work was supported by US DOE grants DE-FG02-97ER54392, DE-FC02-05ER54823 and No. DE-AC-5-00OR22725.

REFERENCES

1. J.-M. Noterdaeme and G. Van Oost, Plasma Phys. Control. Fusion 35, 1481 (1993).
2. J. R. Myra, D. A. D'Ippolito, D. A. Russell, L. A. Berry, E. F. Jaeger, and M. D. Carter, Nucl. Fusion **46**, S455-S468 (2006).
3. J.R Myra, , D.A. D'Ippolito, and M. Bures, Phys. Plasmas **1**, 2890 (1994).
4. D. A. D'Ippolito and J. R. Myra, Phys. Plasmas **13**, 102508 (2006).
5. R. L. Stenzel, Phys. Rev. Lett. **60**, 704 (1988).
6. J. R. Myra, D. A. D'Ippolito, D. W. Forslund and J. U. Brackbill, Phys. Rev. Lett. 66, 1173 (1991).
7. D. A. D'Ippolito and J. R. Myra, Phys. Plasmas **3**, 420 (1996).
8. E. F. Jaeger, L. A. Berry, and D. B. Batchelor, Phys. Plasmas **7**, 3319 (2000).