Analytic Models of Near-Field RF Sheaths

D. A. D'Ippolito and J. R. Myra

Lodestar Research Corporation,
Boulder, Colorado, 80301

June, 2009

Presented at the *18th Topical Conference on Radio Frequency Power in Plasmas*
June 24-26, 2009, Ghent, Belgium

DOE/ER/54392-52 & DOE/ER/86216-6 LRC-08-125

Lodestar Research Corporation

2400 Central Avenue #P-5

Boulder, CO 80301

Analytic Models of Near-Field RF Sheaths

D. A. D'Ippolito and J. R. Myra

Lodestar Research Corporation ,2400 Central Avenue, #P-5, Boulder, Colorado, USA 80301

Abstract. An analytic model is derived for electromagnetic rf wave propagation in a plasma-filled waveguide with rf sheath boundary conditions. The model gives a simplified description of the rf fields and sheath potentials near an ICRF antenna under certain conditions. It assumes an equilibrium magnetic field that is slightly tilted from the normal to the current straps. The rf fields and sheath voltage are calculated in the low-density limit and for the general plasma dielectric using an ordering. The model shows that the launched fast wave couples to a slow wave to satisfy the sheath BC, and the sheath capacitance modifies the relative voltage drops across the plasma and the sheaths. The phasing dependence of the sheath voltage is shown, and a condition is given for validity of the vacuum-field sheath model.

Keywords: fusion, rf sheaths, ICRF antenna, sheath boundary condition, fast wave
PACS: : 52.35.Mw**,** 52.40.Kh, 52.50.Qt, 52.55.Fa

INTRODUCTION

ICRF waves have been used successfully to heat and drive current in many fusion experiments, but optimizing the coupling is often difficult. Launching MW of fast wave (FW) power across the scrape-off-layer (SOL) requires control of the sheath (and other nonlinear) interactions associated with the unwanted but parasitically-coupled slow wave (SW).[1]

RF sheaths have been modeled using a "vacuum-rf-field sheath" approximation, but codes with quantitative predictive capability will require a more self-consistent approach. We have proposed incorporating quantitative sheath calculations into rf codes by using an rf sheath boundary condition (SBC).[2,3] Work is in progress to implement the SBC in the TOPICA antenna code[4] and in other rf codes.

Here, we illustrate its use for antenna sheaths, both as a guide to the physics and as a test case for future benchmarking of antenna codes. The present model illustrates a number of effects of sheath capacitance on the rf field and voltage distribution, and we derive a condition for the validity of the vacuum-rf-field sheath model. The "tenuous plasma" limit ($\varepsilon_\perp = 1,\ \varepsilon_\times << 1,\ \ \varepsilon_\| >> 1$) of this work has already been published;[5] more background and details of the calculation are available in that paper. The present paper describes the generalization of the model[6] to include the full plasma dielectric tensor ($\varepsilon_\perp \sim \varepsilon_\times \sim 1,\ \ \varepsilon_\| >> 1$) allowing treatment of FW launch in arbitrary density plasmas.

THE MODEL

We incorporate the sheath BC into a calculation of electromagnetic wave (coupled FW and SW) propagation in a waveguide filled by a constant density plasma. The model approximates the fields near the front face of a FW antenna, where the density is low but plasma effects can still be important. The FW is launched at x = 0, propagates in the +x direction, and is a standing wave in z. We assume $k_y = 0$, which is valid near the poloidal midplane of an ICRF antenna. The FW is polarized in the y direction. The magnetic field is $\mathbf{B} = B(\hat{e}_z + b_y \hat{e}_y)$ with small field line tilt ($b_y \equiv B_y / B << 1$), and the magnetic field lines intersect conducting walls at $z = \pm L$. Referred to a tokamak, (x, y, z) correspond to the radial, poloidal, and toroidal directions, respectively.

We solve the wave propagation problem using a perturbation expansion with $b_y << 1$, assuming a definite FW parity in z. When $b_y \neq 0$ or $\varepsilon_\times \neq 0$, the launched FW wave also drives a small component with SW polarization ($E_{||} \neq 0$). The amplitude of the SW is chosen such that the total rf field (FW + SW) satisfies the sheath BC. The assumption of definite parity in z is well satisfied at the antenna midplane (y = 0) for a typical two-strap ICRF antenna, with $E_{||}$ having *even* (*odd*) parity in z corresponding to *monopole* (*dipole*) phasing.

RF FIELD SOLUTION

The wave equation is given by $L\mathbf{E} = -(4\pi i / \omega)\mathbf{J}_a$ where $\mathbf{L} = (\mathbf{nn} - n^2\mathbf{I} + \boldsymbol{\varepsilon}) \cdot \mathbf{E}$, and $\boldsymbol{\varepsilon} = \varepsilon_\perp \mathbf{I} + (\varepsilon_{||} - \varepsilon_\perp)\mathbf{bb} + (i\varepsilon_\times / 2)(\mathbf{b} \times \mathbf{I} + \mathbf{I} \times \mathbf{b})$ is the usual plasma dielectric tensor.[5]

Instead of specifying the antenna current, we set $J_a = 0$ and specify the fast wave amplitude E_{0y} at x = 0. For a homogeneous plasma the dispersion relation is given by

$$(\varepsilon_\perp - n_x^2 - n_z^2)(\varepsilon_\perp - n_z^2) - \varepsilon_\times^2 = 0 \quad . \tag{1}$$

When $\varepsilon_\times = 0$, the first (second) bracket gives the FW (SW, for $n_x^2 << \varepsilon_{||}$) dispersion relation. The following BCs[2,3] are imposed at the sheath-plasma interface, z = +L (for solutions with definite parity the BC at z = -L is redundant.):

$$E_x = -\Delta\varepsilon_{||} \frac{\partial E_z}{\partial x}, \quad E_y = -\Delta\varepsilon_{||} \frac{\partial E_z}{\partial y} \to 0 \quad , \tag{2}$$

where $k_y = 0$ was used in the last step. This SBC is derived using the continuity of the normal component of **D** and the tangential components of **E** across the sheath-plasma interface. The terms $\propto k \Delta \varepsilon_{||}$ are due to the sheath capacitance, where Δ is the sheath width in z. In the limit $\Delta \to 0$, we recover the usual "metal wall" BC, viz. that the tangential component of **E** vanishes.

We carry out the solution for $\mathbf{E} = \mathbf{E}_{0f} + \mathbf{E}_{1p} + \mathbf{E}_{1s}$ to first order in b_y.[5,6] In lowest order, we specify the parity of the FW field, $\mathbf{E}_{0f} = \hat{E}_y \cos(k_{zf} z - \delta) \exp(ik_{xf} x)$, where $\delta = 0$ ($\delta = \pi / 2$) for monopole (dipole) phasing. We solve for its polarization and for $k_z = k_{zf}$ subject to the BC that $\mathbf{E}_{0y}(z = L) = 0$. In first order, the FW drives a new field $\mathbf{E}_{1p} \propto b_y$ because the field line is tilted in the direction of $\mathbf{E}_{0f}$. $\mathbf{E}_{1p}$ has SW polarization ($E_{||} \neq 0$) but, being a driven wave, satisfies the FW dispersion relation. The sum $\mathbf{E}_{0f} + \mathbf{E}_{1p}$ does not satisfy the SBC at z = L. Thus, one needs to add a SW field $\mathbf{E}_{1s}$ and determine its amplitude A by the sheath BC, with the result that $A \propto b_y$ and the terms $\propto k_x \Delta\varepsilon_{||} E_z$ in the SBC introduce sheath capacitance effects into the model. To have validity for all x, the SBC requires that $k_x = k_{xf} = k_{xs}$. When the full dielectric tensor is retained, $\mathbf{E}_{1s}$ has classes of terms proportional to b_y and to ε_x.

RF SHEATH SOLUTION

The rf sheath voltage is obtained by integrating $E_{||}$ across the sheath, $V_{sh} \equiv -\int_{sh} dz\, E_z^{(sh)} = -\Delta\, \varepsilon_{||}\, E_z(L)$, where $E_z(L)$ is the field on the plasma side of the sheath-plasma interface. In the *tenuous-plasma* limit[5] and including both phasings, the normalized sheath potential $\hat{V} = V_{sh} / V_{vac}$ is

$$\hat{V} \equiv \frac{-\pi}{2} \frac{V_{sh}\, e^{-ik_x x}}{b_y \hat{E}_y L} = \left(\frac{\pi}{2\eta_0}\right) \frac{(\Delta/L) n_{zf}\ \eta_0 \cos(\eta_0 - \delta)}{\alpha \sin(\eta_0 - \delta) + (\Delta/L)(n_{zf}^2 - 1)\alpha\, \eta_0 \cos(\eta_0 - \delta)} \quad (4)$$

where $\eta_0 \equiv \omega L / c$, $\alpha = 1$ ($\alpha = 2$) for monopole (dipole), and V_{vac} is the vacuum-rf-field sheath potential estimate in monopole phasing (for the lowest mode satisfying the BCs at $z = \pm L$). Equation (4) implies that the SBC solution reduces to the vacuum model result in the limit[5] $(\Delta/L) n_{zf}^2 >> 1$. When the *full plasma dielectric* tensor is retained, the monopole phasing version ($\delta = 0$) of Eq. (4) generalizes to

$$\hat{V} = \frac{\pi}{2\eta_0} \left(\frac{n_{zf}}{n_{zf}^2 - \varepsilon_\perp}\right) \frac{n_x^2 (\Delta/L) \eta_{zs} \cos\eta_{zs}}{(1 - Q_f R_s) \sin\eta_{zs} - n_x^2 (\Delta/L) \eta_{zs} \cos\eta_{zs}} \ . \quad (5)$$

Here $\eta_{zs} \equiv k_{zs} L$, $Q_f = \varepsilon_\times / (\varepsilon_\perp - n_{zf}^2)$, $R_s = \varepsilon_\times (\varepsilon_\perp - n_x^2 - n_{zs}^2)$, n_x satisfies Eq. (1), and only terms $\propto b_y$ were kept in the numerator (valid below a threshold density). Additional terms in the numerator, proportional to $\varepsilon_\times$, will be discussed elsewhere.[6] Note that the FW spectrum allowed by the BC is $n_{zf} = (\pi/\eta_0)(1/2 + n)$, $n = 0,1,2...$.

In Fig. 1, we plot $\hat{V}$ vs density for the lowest (n = 0) mode using the result in Eq. (5). The other parameters are b_y = 0.1, L = 50 cm, f = 45 MHz, B = 30 kG, and T_e = 30 eV. For these parameters, the vacuum-field sheath result is recovered only at very low density ($n_e \leq 10^7\ \mathrm{cm}^{-3}$).

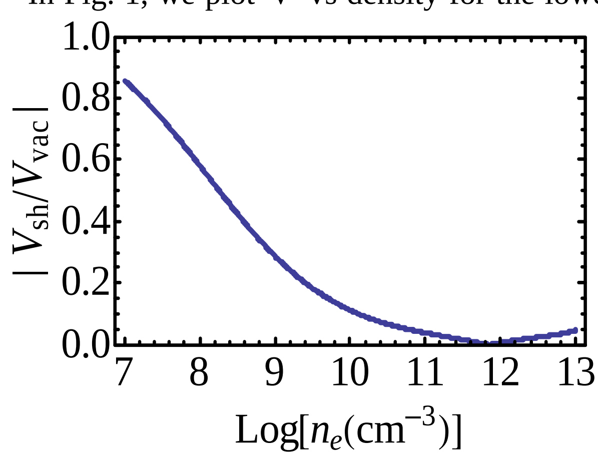

FIGURE 1. Plot of the normalized sheath voltage in Eq. (5) vs density for typical antenna parameters.

A plot using Eq. (4) yields identical results for $n_e \leq 10^{11}\ \mathrm{cm}^{-3}$, where $\varepsilon_\perp << n_{zf}^2$, so in this low-density region the decrease in the sheath voltage with density is due to smaller $\Delta / L \propto \lambda_D / L \propto 1/n_e^{1/2}$. The sheath potential vanishes at the FW cut-off ($n_x^2 = 0$) density. At very high density (not shown in Fig. 1) $\hat{V}$ has a large resonance-like peak, which requires further study. The numerical solution also shows that at fixed density $\hat{V}$ drops rapidly with connection length L. Thus, the approximate vacuum-rf-field sheath model is more likely to apply on field line segments with shorter connections to the sheaths.

In Eqs. (4) and (5) the sheath width is an input parameter, but for self-consistency with sheath theory the Child-Langmuir (CL) Law must be satisfied: $\Delta = \lambda_D (eV_0 / T_e)^{3/4}$ with $\lambda_D = (T_e / 4\pi ne^2)^{1/2}$. Here $V_0 = 3T_e + 0.6V_{sh}$ is the dc sheath potential including thermal and rf sheath contributions. Then Eq. (4) and the CL Law lead to a nonlinear constraint of the form[5]

$$\frac{C \hat{V}^{1/4}}{A - B \hat{V}} = \left(\frac{\lambda_D}{L}\right)\left(\frac{eV_{vac}}{T_e}\right)^{3/4} \tag{6}$$

(Eq. (5) leads to the same constraint with different coefficients.) Solution of this yields a nonlinear curve $\hat{V}(eV_{vac} / T_e)$ with a knee in eV_{vac} / T_e for achieving $\hat{V} \sim 1$.

CONCLUSIONS

This analytic model describes a number of plasma corrections to the vacuum-rf-field sheath model. Most of these corrections result from the sheath capacitance and can be described by treating the sheath as a thin vacuum ($\varepsilon_{sh} = 1$) layer. The model includes the screening of $E_{||}$ from the plasma when $\varepsilon_{||} >> 1$, and the concentration of the voltage drop across the sheaths[5] when $\Lambda \equiv -\varepsilon_{||} \Delta / L >> 1$. The sheath potential calculated here reduces to the vacuum-rf-field result in the limit $(\Delta / L) n_{zf}^2 >> 1$, which is hard to obtain for realistic antenna parameters unless L is short. Finally, the tenuous plasma model is accurate for densities below cut-off in Fig. 1 when $n_{zf}^2 >> \varepsilon_{\perp}$.

ACKNOWLEDGMENTS

We thank the RF SciDAC team for many useful discussions. This work was supported by USDOE grants DE-FG02-97ER54392 and DE-FC02-05ER54823.

REFERENCES

1. J. R. Myra, D. A. D'Ippolito, D. A. Russell, L. A. Berry, E. F. Jaeger, and M. D. Carter, Nucl. Fusion **46**, S455 (2006).
2. J. R. Myra, D. A. D'Ippolito, and M. Bures, Phys. Plasmas **1**, 2890 (1994).
3. D. A. D'Ippolito and J. R. Myra, Phys. Plasmas **13**, 102508 (2006).
4. B. Van Compernolle, R. Maggiora, G. Vecchi, D. Milanesio and R. Koch, *35th EPS Conference on Plasma Phys., Hersonissos, 9 - 13 June 2008 ECA Vol .**32**, P-2.105 (2008).*
5. D. A. D'Ippolito and J. R. Myra, Phys. Plasmas **16**, 022506 (2009).
6. D. A. D'Ippolito and J. R. Myra, manuscript in preparation.