The compact dipole configuration for plasma confinement

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A new magnetic configuration for magnetic confinement of fusion plasmas is proposed. This configuration is closely related to the levitated dipole configuration, and shares the same mechanism for plasma stability. The difference between the two configurations rests in the use of shaping coils to alter the far field region of the dipole field, resulting in greatly reduced total reactor volume given equivalent plasma parameters and core volume. In this paper we will discuss the magnetic geometry of the compact levitated dipole configurations and employ stability theory in the low beta limit to predict its properties.

I. INTRODUCTION

The compact levitated dipole configuration (CLD) is a magnetic configuration for plasma confinement that arises through modifications to the existing levitated dipole configuration [1]. This new configuration shares the topological features of the levitated dipole configuration, as well as mechanical requirements (such as the need for a levitated central coil) and magnetic geometry in the near-field region. The CLD diverges from this simple template by employing shaping coils to radically alter the dipole magnetic geometry in the far-field region. Since the far-field region is the only region in which the levitated dipole configuration resembles the ideal dipole from electromagnetic theory, the result is a configuration that is dipole in name only. More importantly, since the far-field region accounts for most of the volume of the dipole, this alteration results in a configuration that has a much higher ratio of core (near peak density) volume to total volume. It is this key advantage that inspires the name of this new configuration.

In this paper we will discuss the magnetic geometry of the CLD and employ stability theory appropriate to levitated dipoles in the low-beta limit to assess its properties.

II. BACKGROUND

The levitated dipole configuration is a geometry for magnetic plasma confinement that is based on the naturally occurring magnetic fields of stars and planets. The notion that dipole fields could be useful for plasma confinement was first realized by Akira Hasegawa [2]-[3]. He recognized that, unlike most magnetic confinement geometries, magnetic fluctuations in a dipole field can transport particles and energy inward rather than outward. This property gives the dipole a natural resistance to anomalous transport, which gives it considerable potential for confining fusion plasmas.

This potential inspired the construction of the Levitated Dipole Experiment (LDX) as a joint project by the Massachusetts Institute of Technology and Columbia University [1][4][5]. This experiment consists of a superconducting ring inside a toroidal cryostat, suspended inside a vacuum chamber by a second superconducting coil above it. Plasma is formed in the magnetic field region surrounding the levitated ring. Over its duration of funding, this experiment was able to demonstrate many of the predicted properties of the dipole configuration [6]-[7].

While the dipole in its basic form has many appealing properties, there are some areas where there is room for improvement. In particular, one problem with the levitated dipole configuration is its large space requirement. In order to achieve a high peak temperature and pressure, the plasma edge, and therefore vacuum vessel wall, must be separated from the levitated ring by a distance larger than its major radius. This results in a configuration in which the peak pressure region represents only a small fraction of total plasma volume. This presents a potential problem for the economics of a levitated dipole reactor due to the sheer volume of its vacuum vessel; while the proportion of magnet volume to core plasma volume in a levitated dipole is comparable to other plasma confinement schemes, the unusual ratio of reactor volume to magnet volume results in an extremely large reactor.

The goal of the CLD is to solve this problem by rearranging the far-field region of the dipole into a compact flare region in which magnetic field drops rapidly in a limited space. Because magnetic field strength drops rapidly in the flare region, a high critical pressure gradient and therefore a high ratio of core to edge temperature can be achieved. As we will see in Sec. IV., this can be done in a much smaller volume than would be required by a conventional dipole to achieve the same critical pressure differential. Consequently, the need for an expansive vacuum vessel is eliminated.

III. STABILITY THEORY

Ideal magnetohydrodynamic (MHD) stability in a levitated dipole is achieved by two main mechanisms. In the "inward" side of the plasma, between the levitated ring and the peak pressure region, stability is achieved through good magnetic field line curvature. On the "outward" side, between the peak pressure region and the vacuum vessel, however, good curvature does not exist.

In the bad curvature region, plasma stability is achieved through plasma compressibility. To understand this, imagine an interchange mode involving a particular pair of flux bundles. This mode will be unstable if it lowers the total plasma energy and stable if it increases it. Now while the bundle moving outwards loses energy as the plasma adiabatically expands, the bundle moving inwards gains energy as the plasma adiabatically contracts. The energy increase in the inwards moving bundle counteracts the energy decrease of the outwards moving bundle, and contributes to stabilizing the interchange mode. Whether this results in net stability can be determined from the pressure gradient and the flux tube volume[8][9][10]:

$$\frac{d \ln p}{d \ln U} < \gamma \tag{1}$$

where $U = \oint dl/B$ and $\gamma = 5/3$ for a three-dimensional system.

Assuming a pressure profile which is at the marginal stability limit, this yields a formula for the peak/edge pressure ratio:

$$\frac{p_{core}}{p_{edge}} = \left(\frac{U_{edge}}{U_{core}}\right)^{\gamma} \tag{2}$$

In a conventional dipole, this variation in flux tube volume is achieved through field line lengthening and magnetic field fall-off as one progresses from the peak (vicinity of the levitated ring) to the edge. This can be analyzed in two regimes: the near-field region and the far-field region. In the near-field region, the magnetic field can be approximated by that of a straight wire, hence $B \propto r^{-1}$. In the far-field region, the magnetic field can be approximated by that of a dipole, hence $B \propto r^{-3}$. In both cases, the magnetic field line has a length proportional to radius. This results in a scaling of the flux tube volume of $U \propto r^2$ in the near-field region and $U \propto r^4$ in the far-field region. Since pressure scales with flux tube volume as $pU^{\gamma} = const.$, this yields a marginally stable pressure gradient of $p \propto r^{-10/3}$ in the near-field case and $p \propto r^{-20/3}$ in the far-field case.

The peak/edge pressure gradient is essential for confining high-temperature plasmas in the dipole configuration because the edge does not have good confinement, so the smaller the energy density of the edge plasma relative to the core the longer the average energy confinement time which is possible. In addition, if we assume that the plasma achieves a marginally stable gradient by convection due to unstable interchange modes which then bring the pressure gradient back to marginal stability, then the plasma will assume an adiabatic temperature profile $T \propto U^{\gamma-1}$. This means that a high ratio in U will correspond to a hot plasma at the pressure peak, which is essential to fusion.

The question is: what is the most efficient means by which a high ratio in differential flux tube volume can be achieved?

IV. COMPACT DIPOLE CONFIGURATION

As we saw in Sec. III., the conventional levitated dipole configuration achieves a high contrast in flux tube volume mainly through the fall-off of magnetic field strength in the far-field region. This achieves such contrast at a considerable expense in volume, since total volume will scale as $V \propto r^3$. This means that peak/edge pressure ratio will increase with volume at a scaling law barely greater than 2. A more efficient design can be achieved by finding a way to achieve a fall-off in magnetic field strength without adding substantially more volume to the device.

This alternate method of achieving magnetic field fall-off is to constrain the flux at the edge of the device to a cylindrical region centered around the device's axis of symmetry. Assuming the net flux through the cylinder is zero, the magnetic field strength will decrease exponentially as one advances along the cylinder. This decrease in the magnetic field creates a corresponding increase in flux tube volume. The amount of additional volume added to the device scales logarithmically with the desired increase in flux tube volume, which in turn means that a given increase in flux tube volume can be achieved with a relatively insignificant increase in device volume.

There is a practical limit to this approach, which is determined by the relative scaling of magnetic and plasma pressure. As one progresses along the cylinder, the effective length (flux tube volume times field strength) of each field line remains roughly constant due to self-similarity. This means that plasma pressure at marginal stability will scale as $P_{plasma} \propto B^{5/3}$, whereas magnetic pressure scales as $P_B \propto B^2$. This results in a net increase in beta, which scales as $\beta \propto B^{-1/3}$.

A moderately high edge beta can be accepted because the x-point acts as a cusp mirror. In the cusp mirror configuration, ideal MHD stability is achieved through good curvature everywhere. The drawback of this configuration is poor confinement along open field lines. Poor confinement is acceptable in this case because the plasma pressure in the open field-line region required for stability is extremely small compared to the peak, and thus contains only a tiny minority of the total energy of the configuration. However, there is still going to be a practical limit on the plasma pressure which can be confined in the edge pedestal. For purposes of developing scaling principles, let us suppose that an acceptable operating regime employs an edge beta comparable to the beta at the pressure peak.

In this case, the maximum ratio of peak/edge flux volume is determined by the aspect ratio of the levitated ring. In the near-field region, the magnetic field only drops off as 1/r, yielding $U \propto r^2$. This gives $P_B \propto r^{-2}$ and $P_{plasma} \propto r^{-10/3}$ which is a net beta drop of $\beta \propto r^{-4/3}$. Since in the cylinder region $\beta \propto B^{-1/3}$, the amount of magnetic field falloff we can achieve in the cylinder region is proportional to the aspect ratio of the ring to the fourth power. Given an aspect ratio a, this yields a ratio:

$$\frac{U_{edge}}{U_{core}} \propto a^6 \tag{3}$$

Due to the sixth power dependence in this formula, only a modest aspect ratio on the levitated ring is required to achieve an extraordinary peak/edge pressure ratio. Consequently, the ratio of total device volume to the volume of the peak pressure region can be quite modest for a device capable of high core pressure at low edge pressure. Total volume in this type of device scales roughly as the major radius of the ring with minor logarithmic scaling as the aspect ratio increases. Peak volume scales roughly as the inverse square of aspect ratio. Thus, the scaling of peak/edge pressure ratio with total/peak volume ratio is roughly a fifth power proportion. This is significantly better than the roughly quadratic proportion we saw with the conventional dipole. We expect then that a significant reduction in volume will be realized once such a configuration is implemented.

Unfortunately, at present no systematic means exists to design magnetic configurations which satisfy the above conditions. Fortunately, reasonable geometries can still be developed by a trial-and-error approach. The results of this approach are discussed in the following section.

V. RESULTS

In order to calculate the performance of the device described above, we employ a computer program to evaluate the magnetic field of a collection of ring currents[11]. We can then calculate the flux tube volume along various flux surfaces, ranging from the surface of the levitated ring to the vicinity of the separatrix. We can also calculate the minimum magnetic field along a flux surface, which allows us to calculate whether a particular solution satisfies our conditions for a realistic edge beta. We also calculate the volume enclosed by each flux surface, which permits an estimation of the space requirements of a configuration relative to its core volume. These results can then be compared to those for a free dipole.

To make a comparison between the conventional levitated dipole and the compact levitated dipole, we employ two configurations. One is a simple ring current, which we will use to calculate properties of the conventional dipole. The other consists of a levitated ring flanked by five eternal coils. Each of these external coils carries a current which is .0775 times the levitated ring current and in the opposite direction. Due to symmetry this applies no net force on the levitated ring, so some adjustments will be necessary to provide for vertical support, however since we have not specified the weight of the ring or the strength of the magnetic field we leave this configuration in a symmetrical arrangement for simplicity of calculation. The flux contours which result from this arrangement of currents is shown in Fig.1.

Given this configuration, we can then calculate the flux tube volume by integrating along specified flux surfaces. Once we have the flux tube volume, we can calculate relative pressure by taking that to the 5/3 power, and maximum beta can be calculated by dividing pressure by the square of the minimum magnetic field along that flux surface. These results are shown in Fig. 2. For comparison, a levitated dipole with roughly equivalent performance produces the results in Fig. 3.

In terms of peak/edge pressure ratio, these results are comparable, as they were selected to be. However, in the case represented in Fig. 3, the central flux surface encloses a volume of 1.59 cubic units, where the unit in this scale is set to the major radius of the central ring. The outermost flux surface encloses a volume of 474.28 cubic units. This is a ratio of 298:1. With the compact dipole, on the other hand, the central flux surface only encloses a volume of .69 cubic units even though it is at the same flux coordinate. This is probably due to the squeezing effect of the external coils on the magnetic field on the outboard side of the levitating ring, which would otherwise fan out significantly. The outermost flux surface encloses a volume of 37.16 cubic units, a ratio of 54:1. This represents a nearly sixfold decrease in total volume relative to the volume of the core region, with no appreciable compromise in peak/edge pressure ratio. The only price paid, other than the addition of the external coils, is a rise in beta towards the edge, which for reasons previously discussed is within acceptable levels for this sample geometry.

VI. CONCLUSIONS

The compact levitated dipole is a variant on the levitated dipole configuration which offers a high peak/edge temperature ratio in a considerably smaller space than an equivalent conventional levitated dipole. This is achieved through the use of external coils to confine far-field magnetic flux to a roughly cylindrical region in which it its strength falls off rapidly.

The main drawback of the CLD is the increase in beta near the edge. This can be tolerated if the edge has good MHD stability properties, however since tolerable edge beta will be finite this presents a constraint on reactor design. Additional stabilizing effects might arise due to FLR effects, particularly in light of the extreme degree of magnetic fall-off at the edge of the CLD which might put ion Larmor radii above device scale for certain plasma parameters. Such an effect would eliminate the need for a pressure pedestal at the separatrix, however, this possibility is not investigated fully in this paper. In either case, a more detailed stability calculation at the edge is needed to accurately estimate the potential of this type of device.

The most important result is the realization that adjustments in the magnetic geometry of a levitated dipole can produce significant improvements in geometric parameters relevant to reactor economics. This type of design approach allows the creation of levitated ring devices with better plasma parameters while making compromises at considerably scaling laws than are possible with a conventional dipole. This will result in smaller, more convenient potential reactor designs and thus improves the long-term viability of the dipole concept as a whole.

References

[1] D. T. Garnier *et al*, Fus. Engr. and Design **81**, 2371 (2006).

[2] A. Hasegawa, Comments Plasma Phys. Control. Fusion 1, 147 (1987).

[3] A. Hasegawa, L. Chen, and M. Mauel, Nucl. Fusion 30, 2405 (1990).

[4] J. Kesner, L. Bromberg, D. Garnier, and M. Mauel, 17th IAEA Fusion Energy Conference, Paper No. IAEA-F1-CN-69-ICP/09 Yokohama, Japan (International Atomic Energy Agency, Vienna, 1998).

[5] LDX white paper, J. Kesner, L. Bromberg, M. Mauel, D. Garnier, J. M. Dawson (1998).

[6] D. T. Garnier *et al*, Nucl. Fusion **49**, 055023 (2009).

[7] D. T. Garnier *et al*, Phys. Plasmas **13**, 056111 (2006).

[8] I.B. Bernstein, E. Frieman, M. Kruskal and R. Kulsrud, Proc. R. Soc. Lond. A 244 17 (1958).

[9] M.N. Rosenbluth and C.L. Longmire, Ann. Phys. 1, 120 (1957).

[10] J. Kesner, PSFC report

[11] Classical Electrodynamics, 3rd ed., J.D. Jackson p. 182

Figure captions

Figure 1: Flux surfaces for a compact levitated dipole. The flux surfaces shown correspond to flux coordinates of .00001, .0001, .0003, .001, .003, .01, .03, .1, .3, 1, 2, and 3, respectively. Black circles represent positions of field coils.

Figure 2: Relative plasma pressure and relative beta as a function of flux coordinate for a compact levitated dipole.

Figure 3: Relative plasma pressure and relative beta as a function of flux coordinate for a conventional levitated dipole.

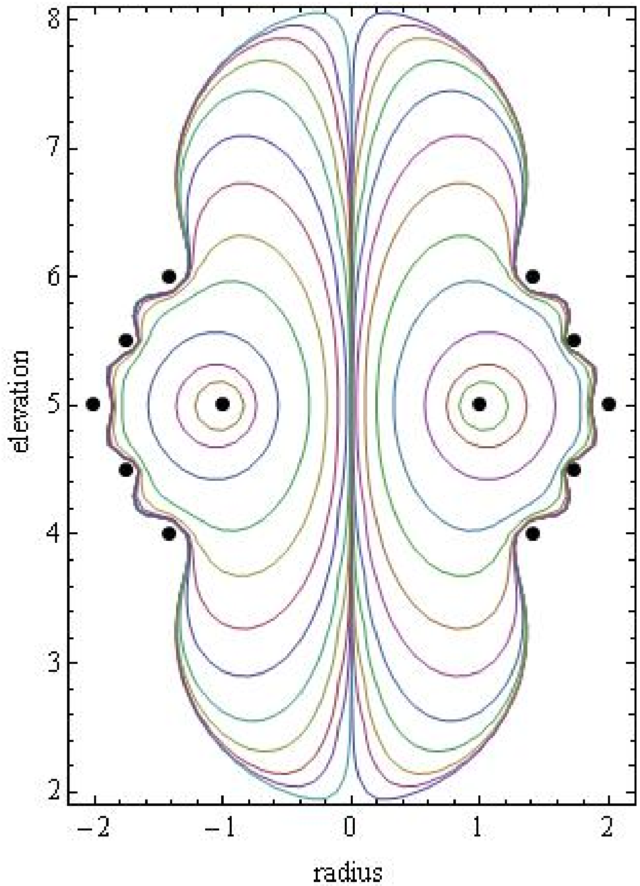

Figure 1:

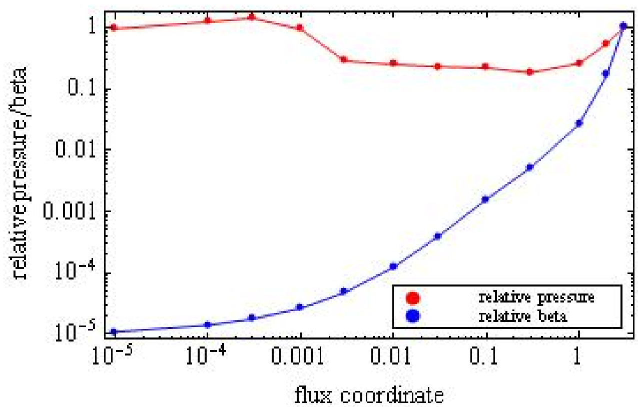

Figure 2:

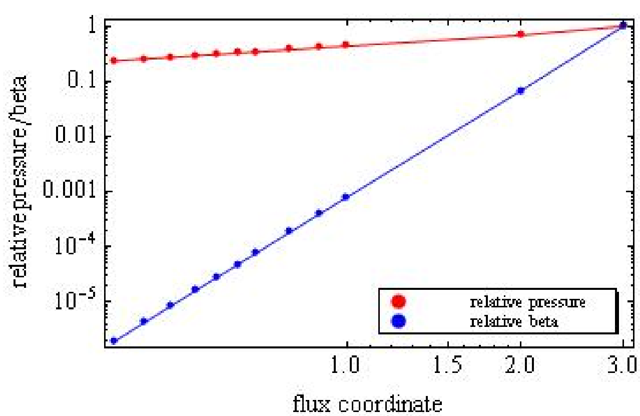

Figure 3: