Role of Magnetic Field Tangency Points in ICRF Sheath Interactions

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June 2013

presented at
20thTopical Conference on Radio Frequency Power in Plasmas
Sorrento, Italy, June 25-28, 2013

DOE-ER/54392-73; DOE-ER/54823-16 LRC-13-155

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Abstract. ICRF waves can sometimes interact with plasma-facing surfaces in tokamak fusion experiments causing degradation of core heating efficiency, impurity injection and even component damage. While presently available low dimensionality rf sheath models are useful in understanding many features of these interactions, more quantitative modeling will require attention to realistic geometrical details of the boundary plasma and surfaces. In this paper, we explore the situation in which there exists a tangency point of the background magnetic field with a surface. We find that the rf interactions are strongly influenced by the generation and propagation of sheath-plasma waves (SPW) along the surface. It is found that these waves preferentially propagate towards, and accumulate at, a convex tangency point. An analytical theory of SPW propagation is developed to understand these features.

INTRODUCTION

Waves in the ion cyclotron range of frequencies (ICRF), routinely used for heating and current drive in fusion experiments, can interact with plasma-facing surfaces, sometimes causing detrimental effects. In the worst cases, degradation of core heating efficiency, impurity injection and even component damage can occur. These effects, are often attributable to rf-sheath interactions [1-6] which are ultimately driven by waves with a parallel component of rf electric field, $E_{||}$.

Strong rf-sheaths can be formed on the antenna Faraday shield, side walls and septa. There is an active program to control these types of sheaths by careful antenna design.[2] Wave propagation characteristics in the scrape-off layer (SOL) may cause some fraction of the launched wave power to propagate directly into limiters or other plasma facing surfaces,[7] providing another opportunity for rf-sheath interactions. Finally, wave power that reaches the core and is not absorbed can be reflected back to the SOL[1] or be transmitted through the core to the opposite (high field side) SOL where additional surface interactions can occur. These types of interactions are minimized when single pass absorption is good.

In each of these cases, the background magnetic field lines intercept the surface at some angle, which may vary along the surface and rf wave energy and plasma coexist at some location along that surface. It would be natural to expect the strongest rf sheath interactions to occur where the wave amplitude (in particular $E_{||}$) maximizes on the surface, but we will show here that local plasma and geometry considerations can also play an important role.

In simple models the sheath may be treated as a vacuum region of width $\Delta \sim \lambda_d$ the Debye length. This is because the parallel electron conductivity is dramatically reduced in the non-neutral electron poor sheath region. In this capacitive limit, the effect of the sheath on the bulk plasma waves may be modeled using a sheath boundary condition (BC).[8]

The sharp change in dielectric characteristics between the sheath and the bulk quasi-neutral plasma gives rise to a type of surface wave, called the sheath-plasma wave (SPW).[9] Without the sheath, it could not exist, because the rf fields would normally vanish on the conducting surfaces. The sheath-plasma interface, however, allows capacitive electron charge. The SPW plays a central role in the present work because it can propagate along the surface carrying rf wave energy to places where one might not expect.

The interaction of rf waves with an rf sheath has been studied using a numerical code called rfSOL.[5] The rfSOL code implements the cold plasma rf wave model, plasma density profiles, an arbitrary (spatially varying) background magnetic field, a model antenna, and a bounding surface on which the sheath BC is imposed. The present work is motivated by a numerical result obtained with rfSOL and shown in Fig. 1. In the figure, we see a slow wave (SW) launched from the antenna and propagating along the magnetic field lines to a surface. (While the antenna used here is very simple, realistic nominally fast wave (FW) antennas inevitably exhibit some coupling to the SW, in spite of deliberate attempts to minimize $E_{||}$ by antenna design.) The resulting $E_{||}$ appears to propagate preferentially in one direction along the surface. In the following, we study this interaction using analytic methods. Details of the simulation itself will be presented elsewhere.

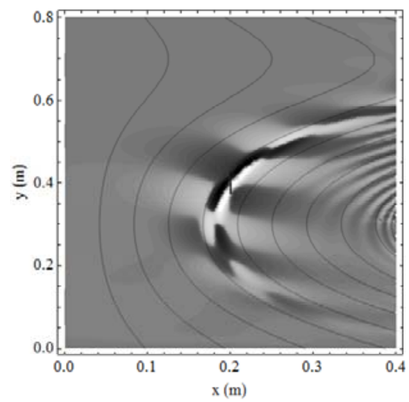

FIGURE 1. Magnetic flux surface contours (thin solid lines) and distribution of rf $E_{||}$ (shaded palette) from an rfSOL simulation. The launcher is located in the center of the figure and k_z (into the page) is specified. The SW approximately follows the field lines to the sheath surface at right impacting near (x,y) = (0.4, 0.6) where short scale SPWs are generated. These SPWs propagate along the sheath surface (downward in the figure) until they encounter the convex tangency point at (x,y) = (0.4, 0.3).

DERIVATION OF THE SPW DISPERSION RELATION

A dispersion relation for the SPW can be derived by matching the solution of the wave equation in the quasi-neutral plasma volume to the solution in the non-neutral sheath region, modeled here as a vacuum gap of width Δ. Equivalently, the sheath BC may be employed. The technique has been illustrated in the electrostatic approximation in previous work.[5] Here, we consider a more complete electromagnetic (EM) treatment.

The SPW is a short scale-length mode, closely related to the SW because of the importance of $E_{||}$ in the wave-sheath dynamics. Thus, in the plasma volume, the waves must obey the EM dispersion relation for the SW given by

$$n_\perp^2 \varepsilon_\perp + n_{||}^2 \varepsilon_{||} = \varepsilon_\perp \varepsilon_{||} \tag{1}$$

where $\mathbf{n} = \mathbf{k}c/\omega$ is the index of refraction, ⊥ and || indicate perpendicular and parallel orientations with respect to the background field direction $\mathbf{b} = \mathbf{B}_0/B_0$. In Eq. (1) $\varepsilon_\perp$, $\varepsilon_{||}$ are components of the cold-fluid dielectric tensor

$$\boldsymbol{\varepsilon} = (\mathbf{I} - \mathbf{bb})\varepsilon_\perp + \mathbf{bb}\varepsilon_{||} + i\mathbf{b}\times\mathbf{I}\varepsilon_\times \tag{2}$$

Other standard notations are defined in Refs. 5 and 8.

The sheath BC is usually expressed as a two-component vector equation in the surface[8] $\mathbf{E}_t = \nabla_t \Delta D_n$ where t and n denote tangential and normal to the surface, respectively, and $\mathbf{D} = \boldsymbol{\varepsilon}\cdot\mathbf{E}$ is the displacement vector. Equivalently, the sheath BC may also be written in the form[10]

$$\nabla_t \cdot \mathbf{E}_t = \nabla_t^2 \Delta D_n \tag{3}$$

$$B_n = 0 \tag{4}$$

where B_n is the normal component of the rf magnetic field at the surface.

In order to apply the sheath BC to the SW in the plasma volume, it is necessary to extract the polarization of the SW, i.e. the E_t, D_n and B_n components. To this end, we recapitulate the derivation of Eq. (1). With mode phases ~ $exp(i\mathbf{k}\cdot\mathbf{x}-i\omega t)$, electrostatic potential Φ and parallel vector potential $A_{||}$, we have the SW field

$$\mathbf{E} = -i\mathbf{k}\Phi + (i\omega/c)\mathbf{b}A_{||} \tag{5}$$

Using the EM wave equation $(\mathbf{nn} - n^2\mathbf{I} + \boldsymbol{\varepsilon})\cdot\mathbf{E} = 0$ one obtains

$$\boldsymbol{\varepsilon}\cdot\mathbf{n}\Phi-(\mathbf{nn}-n^2\mathbf{I}+\boldsymbol{\varepsilon})\cdot\mathbf{b}A_{||}=0 \tag{6}$$

Two components of Eq. (6) may be taken to obtain two independent equations for Φ and $A_{||}$. A convenient set of equations results from taking the scalar product of Eq. (6) with both **n** and **b**. Substituting the explicit form of the dielectric tensor, Eq. (2), yields

$$(n_\perp^2\varepsilon_\perp+n_{||}^2\varepsilon_{||})\Phi-n_{||}\varepsilon_{||}A_{||}=0 \tag{7}$$

$$\varepsilon_{||}n_{||}\Phi+(n_\perp^2-\varepsilon_{||})A_{||}=0 \tag{8}$$

Eqs. (7) and (8) yield the SW dispersion relation given in Eq. (1) and also the desired polarization information.

In electrostatic theory, Eq. (4) is automatic, and the SPW dispersion relation results from the simultaneous solution of Eqs. (1) and (3) in the electrostatic limit. Given ω and k_z, this results in two equations for two unknowns, k_x and k_y. In EM theory there is no exact SPW eigenmode in 2D geometry; however, a quasi-mode SPW can still be driven: the approximate dispersion relation is obtained by neglecting Eq. (4). Then Eq. (3) is cast into the form

$$n_tk_t\Phi-\mathbf{k}_t\cdot\mathbf{b}_tA_{||}=ik_t^2\Delta\mathbf{s}\cdot\boldsymbol{\varepsilon}\cdot(\mathbf{n}\Phi-\mathbf{b}A_{||}) \tag{9}$$

where **s** is the unit vector normal to the surface pointing into the plasma. Exploiting the large size of the parallel dielectric response relative to the other components, we approximate $\mathbf{s}\cdot\boldsymbol{\varepsilon}=\mathbf{s}\cdot\mathbf{bb}\varepsilon_{||}=b_n\varepsilon_{||}\mathbf{b}$ where b_n is the component of **b** normal to the surface. Finally, combining Eqs. (1) and (8) to obtain $\Phi=A_{||}n_{||}/\varepsilon_\perp$ and using this result in Eq. (9) one arrives at

$$n_t^2k_{||}-\varepsilon_\perp\mathbf{k}_t\cdot\mathbf{b}_t=ik_t^2\Delta b_n\varepsilon_{||}(n_{||}^2-\varepsilon_\perp) \tag{10}$$

APPLICATION TO SLOW WAVE SHEATH INTERACTION

When a wave in the plasma volume impinges on a sheath surface, as shown in Fig. 1, the original wave must distort in order to conform to the sheath BC on the surface. This interaction is what drives the short spatial scale SPWs. Note that Eq. (4) for B_n must ultimately be satisfied by the superposition of the SPW waves together with the incoming driving wave (here a SW). This provides an SW-SPW coupling mechanism that is treated exactly in the rfSOL solution. In the following we obtain the approximate local SPW dispersion solution for the parameters of Fig. 1 and show that it describes the observed simulation modes, and can explain the preferential propagation direction of the SPW along the surface, as well as its accumulation at a particular tangency point.

Parameters for this case are: f = 80 MHz, n_e = 2.0×10^{12} cm^{-3}, T_e = 10 eV, B_z = 4 T, k_z = 0.108 cm^{-1}. The poloidal magnetic field (in the x-y plane of Fig. 1) has a maximum amplitude B_{p0} = 0.4 T, and the plasma is deuterium. Note that there are two tangency points of the magnetic field and the sheath surface at y = 0.3 m and y = 0.7 m, which we refer to as the convex and concave tangency points respectively. At these points b_n = 0 and also $\Delta(y)\to 0$ since at grazing angles, the sheath is no longer electron poor (parallel electron losses are small).

Numerical solution of Eqs. (1) and (10) reveals a total of eight complex roots for k_x and k_y, most of which are unphysical. Three of the roots have $Im(k_x)>0$ and must be discarded because with our $exp(i\mathbf{k}\cdot\mathbf{x})$ phase convection; they correspond to modes which grow exponentially as x decreases from the sheath surface into the plasma volume. The other five roots decay away from the sheath and are therefore candidates for SPWs. However, of these five, only two have short wavelengths which satisfy the assumed SW ordering. The corresponding solutions for k_y and v_{gy} along the sheath surface are shown in Fig. 2. Here $v_{gy}=\partial\omega/\partial k_y$ is the group velocity.

Both modes are weakly evanescent in y, $Im(k_y)<<Re(k_y)$, except possibly near the tangency points. They are approximately electrostatic since $|\varepsilon_\perp|/n_{||}^2\sim 0.3<<1$ (not shown). The wavelength in the y-direction at y = 0.40 m is about $2\pi/250$m^{-1} ~ 2.5 cm which is close to the rfSOL wavelength along the sheath surface which we estimate as 2.6 cm. The arrows in the figure indicate the direction of energy propagation, i.e. the group velocity, for each mode.

By considering the sign of $Im(k_y)$, one can see that for one of the modes (indicated by the heavier lines and the label "1") the wave decays in y as it propagates in the v_{gy} direction. For the other mode (indicated by the lighter lines and the label "2") the opposite is true: it grows in the direction of energy propagation. In this local wave theory, there is no apparent source of free energy to drive instability. We conclude that only mode "1" is physical.

The physical mode "1" possesses the property of a preferential direction of propagation in each region which can be summarized in a simple way: the SPW propagates towards the convex tangency point at y = 0.3 m and away from the concave tangency point at y = 0.7 m. Thus the SPW can distribute rf wave energy and concomitant sheath interactions along the surface, and can furthermore cause energy to accumulate at certain tangency points. (Note the slowing of the group velocity near y = 0.3.)

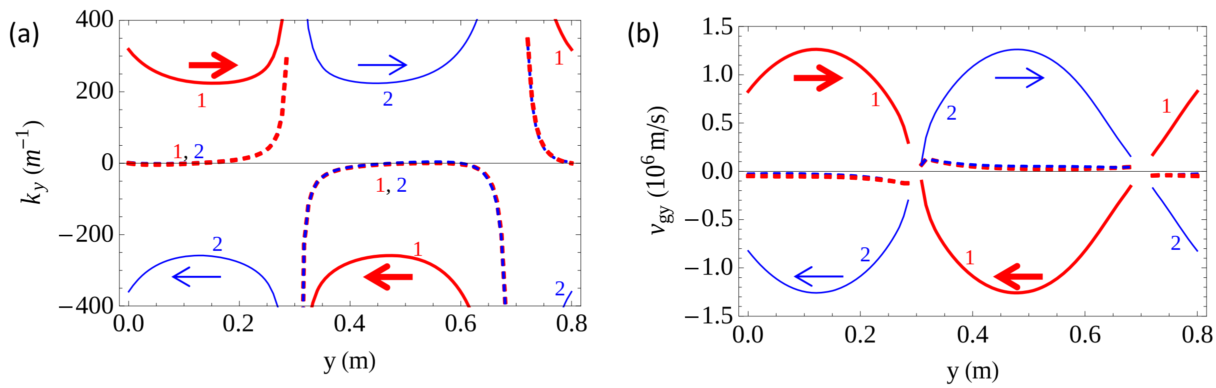

FIGURE 2. Solution of Eqs. (1) and (10) for the parameters of Fig. 1. *Re* and *Im* parts are shown in solid and dashed lines respectively for modes "1" and "2". Arrows indicate the direction of propagation.

DISCUSSION AND CONCLUSIONS

We postulate that the degree to which incident waves (be they SW or FW without any SPW) and the background surface sheath BCs fail to conform is what excites the SPW. When the variation of wave energy or BCs along the sheath surface has a short scale length comparable to the short SPW wavelength, SPWs are strongly excited. Since BC variation is always rapid near the tangency points of the magnetic field with the surface, these are prime locations for strong wave-sheath interaction.[1,4] Furthermore, we have shown that the physical SPW branch has a preferential direction of propagation that depends on the orientation of the magnetic field with respect to the surface normal. For the case considered this preferential propagation direction is always towards the convex tangency point and implies an accumulation of SPW wave energy and sheath interaction at that point. This important result agrees with the rfSOL simulations and is expected to be an important general consideration in interpreting experimental results which show localized "hot spots" for rf-sheath interactions.

ACKNOWLEDGMENTS

The authors gratefully acknowledge discussions with the RF SciDAC Team (Center for Simulation of Wave-Plasma Interactions). This work was supported by grants DE-FC02-05ER54678 and DE-FG02-97ER54392 with the US DOE.

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