Slow Wave Propagation and Sheath Interaction for ICRF Waves in the Tokamak Scrape-off-Layer

J.R. Myra and D.A. D'Ippolito

Lodestar Research Corp., Boulder, Colorado USA

acknowledgement: RF SciDAC Team

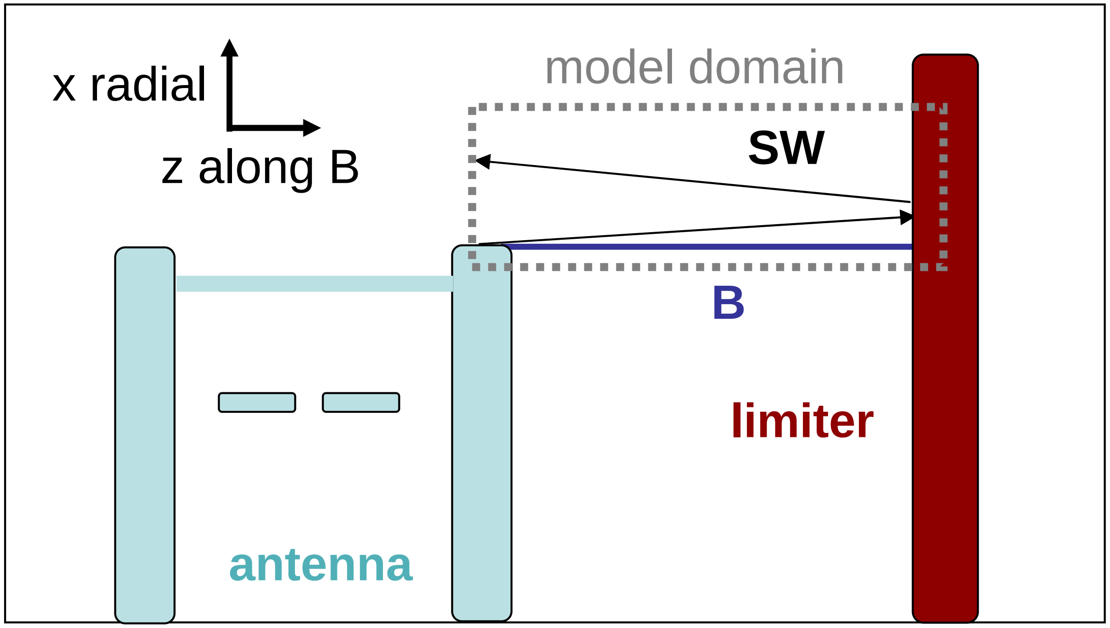
presented at the 18th Topical Conference on Radio Frequency Power in Plasmas
24 - 26 June 2009 Gent, Belgium

Work supported by USDOE grants DE-FG02-97ER54392
and DE-FC02-05ER54823 (the RF SciDAC project)

Motivation and background

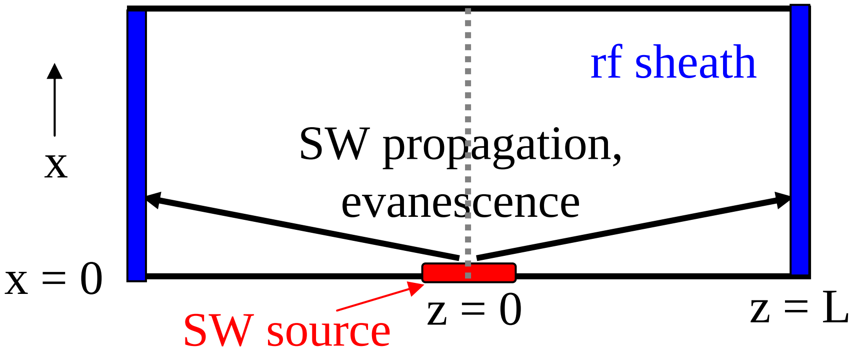
- interested in ICRF sheath interaction with walls and limiters
  - sputtering, impurities, power loss, …
- rf sheaths generated primarily by the $E_{||}$ component ⇒ slow wave (SW)
- previously studied situations in which the FW can access the wall directly
  - poor central absorption or surface wave phenomena
  - FW generates SW to satisfy BC when wall normal has a component along B
    - J. R. Myra, D. A. D’Ippolito and M. Bures, Phys. of Plasmas **1**, 2890 (1994).
    - D.A. D'Ippolito, J.R. Myra, E.F. Jaeger and L.A. Berry, Phys. Plasmas **15**, 102501 (2008).
- also have studied sheaths on the antenna structure
  - B-field tilt wrt. current strap ⇒ SW generation [D'Ippolito PoP 2009; & this meeting]
- now consider case where SW is generated at antenna and propagates/evanesces into SOL
  - this poster: model problems with simple rectangular geometry and constant density plasma ⇒ seek concepts and insight
  - in progress (H. Kohno et al.): numerical solution of SOL propagation with sheath BCs in a realistic SOL geometry ⇒ quantitative prediction

Geometry of Slow Wave (SW) excitation



- SW components, i.e. $E_{||}$, generated at antenna: e.g. protection tips and possibly at side-wall gaps (from B-field misalignment)
- SWs generated from localized source in z and x
- understand their propagation & interaction with limiter sheaths
- Que: *How much of the antenna sheath voltage appears across limiter sheaths; How much is dropped across the plasma?*

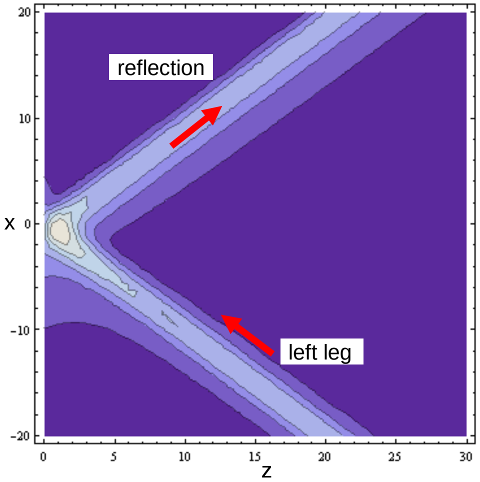
Model geometry – the aperture problem

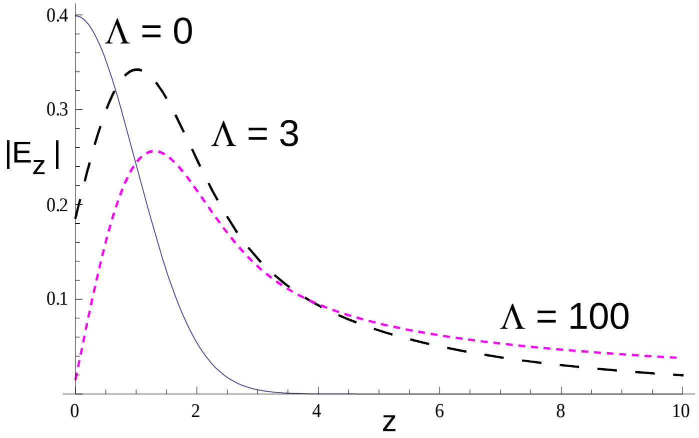


- SW is emitted through an aperture (source) into a box (SOL) bounded by conducting walls (limiters)
- study the propagation/spreading/evanescence of the SW and its interaction with wall sheaths
- previous work (PRL 2008): the tenuous plasma limit $n_e < n_{lh}$ ($\omega > \omega_{lh}$)
  - SW propagates as resonance cone (RC) without spreading
  - reflects off of wall and generates self-consistent rf sheath
- present work: the dense plasma limit $n_e > n_{lh}$ ($\omega < \omega_{lh}$)
  - SW is normally evanescent, but here we will see it is not

Review: tenuous plasma limit: Resonance Cones (RCs)

- tenuous plasma model $\overleftrightarrow{\varepsilon} = (\overleftrightarrow{I} - \mathbf{bb}) + \mathbf{bb}\varepsilon_{||}$
- EM SW dispersion relation $n_x^2 = -\varepsilon_{||}(n_z^2 - 1)$
- ES limit $k_x = -(\omega_{pe} / \omega)|k_z|$
- sheath BC (vacuum gap Δ) $E_x \mp \partial_x \Delta\varepsilon_{||} E_z = 0$
- use method of images to construct solution satisfying sheath BC
- key parameter is $\Lambda_{RC} = -\dfrac{\Delta\varepsilon_{||}}{a_{||}}$ $a_{||}$ = parallel scale-length of RC

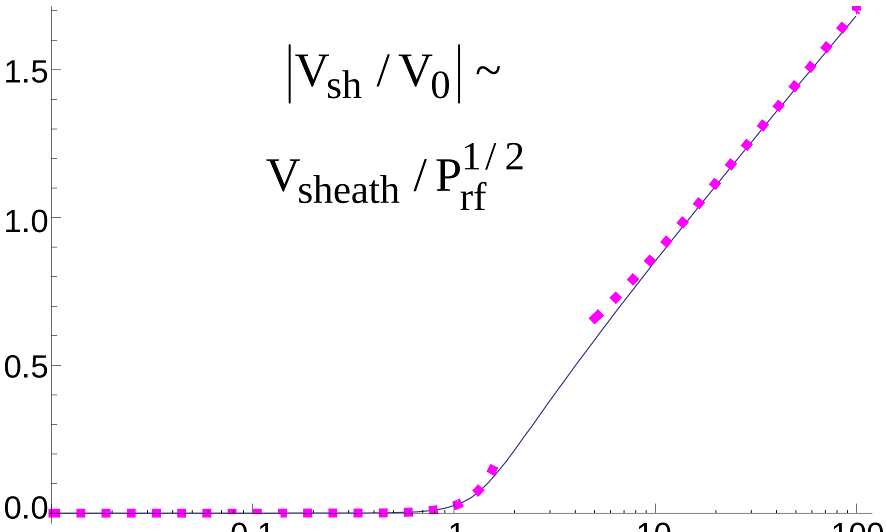




Tenuous plasma (cont'd):

RC Sheath voltage transmission for self-consistent Δ shows a threshold at $\Lambda_0 \sim 1$ - 4

J.R. Myra and D.A. D'Ippolito, Phys. Rev. Lett. **101**, 195004 (2008).



$$\Lambda_0 = -\frac{\lambda_{de}\varepsilon_{||}}{a_{||}}\left|\frac{\alpha e V_0}{T}\right|^{3/4} \sim P_{rf}^{3/8} n_e^{1/2}$$

- use Child-Langmuir law: make Δ consistent with fields at wall

$$\Delta = \lambda_{de}\left|\frac{\alpha e V_{sh}}{T}\right|^{3/4}$$

- estimates for C-Mod show $\Lambda_0 \sim 4$ occurs when RC structures ~ 200 V are launched with parallel scale $a_{||} < 15$ cm.

High density SOL: model problem

- constant density plasma
- symmetric: consider modes even in $E_{||}$
- local SW dispersion relation in plasma region $\varepsilon_\perp n_x^2 = \varepsilon_{||}(\varepsilon_\perp - n_z^2)$
- sheath BC at wall , z = L $E_x = -ik_x \Delta\varepsilon_{||} E_z$
- determines a global dispersion relation ⇒ eigenmodes of box

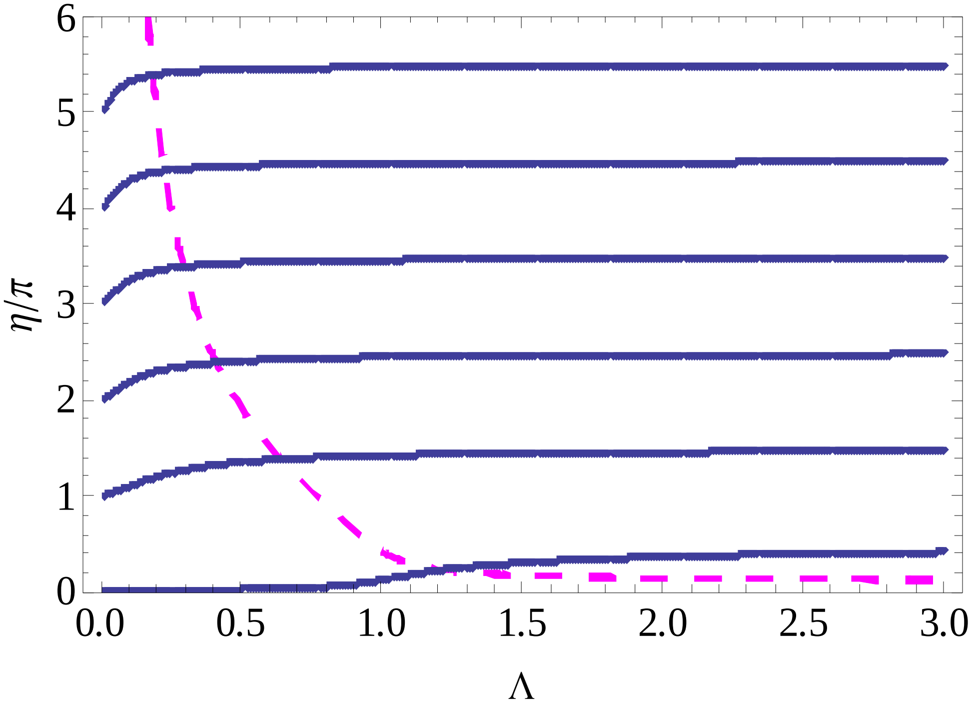
$$\boxed{\eta \tan\eta = (\eta^2 + b^2)\Lambda} \quad \text{where} \quad \begin{array}{l} \eta = k_z L \\ \eta_0 = \omega L / c \end{array} \quad b^2 = -\varepsilon_\perp \eta_0^2 \quad \Lambda = -\frac{\Delta\varepsilon_{||}}{L}$$

- procedure:
  - first find a complete set of eigenfunctions for the box that satisfy sheath BCs in z and are outgoing/evanescent in x
  - expand the source (at x = 0) in this basis set
  - summed eigenfunction behavior determines solution at x > 0

Eigenfunctions of the box

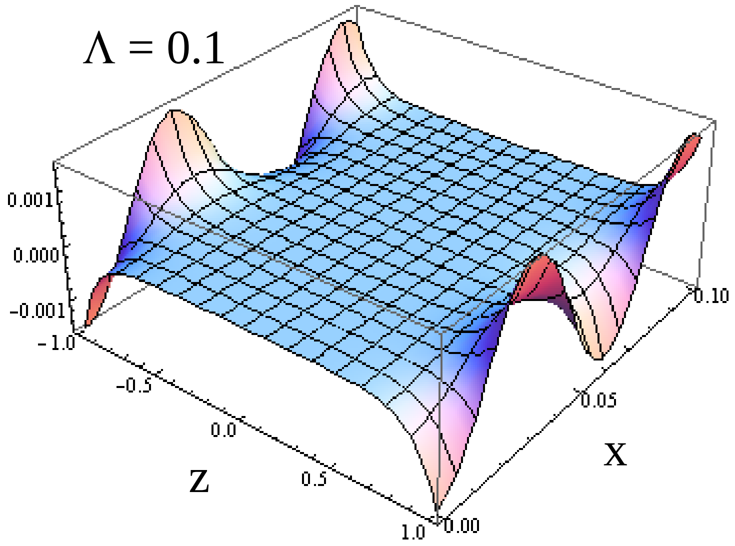
$$\boxed{\eta \tan \eta = (\eta^2 + b^2)\Lambda}$$

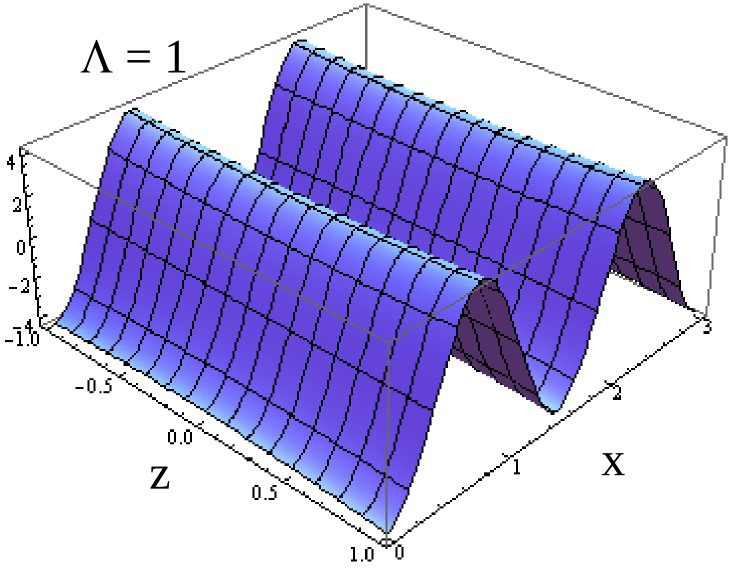
- $\Lambda = 0$ is metal wall limit $\Rightarrow \cos(k_m z)$ with $\eta_m = m\pi$, $m = 0, 1, 2, \ldots$
- $\Lambda = \infty$ is insulating limit $\Rightarrow \cos(k_m z)$ with $\eta_m = m\pi/2$, $m = 1, 3, 5 \ldots$
- intermediate Λ roots transition, but there is also a new root with pure imaginary $\eta \Rightarrow$ sheath-plasma wave (SPW)



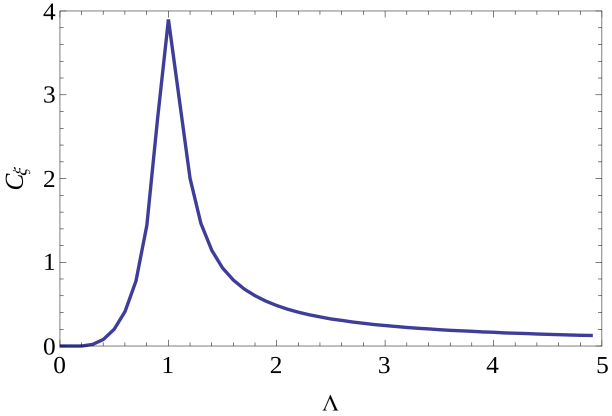
Roots for b = 0.1,
Re (blue),
Im (dashed magenta).

SPW can be localized to sheaths or global





- field pattern $\mathrm{Re}[E_z(x,z)]$ for the SPW eigenmode (imaginary root)
- $\Lambda << 1 \Rightarrow \mathrm{Im}(\eta) > 1 \Rightarrow$ mode hugs the sheath boundary [see also D'Ippolito PoP 2006, Myra PRL 1991]
- projection of localized source onto SPW is small for $\Lambda << 1$ or $\Lambda >> 1$



Projection C_ξ onto the SPW vs. Λ

- recover metallic and insulating complete sets for $\Lambda << 1$ or $\Lambda >> 1$, without the SPW.
- note SPW resonance near $\Lambda = 1$.

Insights from the metal wall limit Λ = 0

$$E_z = \sum_m C_m \cos k_{mz} z \, e^{i k_{mx} x}$$

at x = 0: $\sum_m C_m \cos k_{mz} z = \delta(z)$

- $b \gg 1 \Rightarrow$ evanescence on the scale $x \sim \delta_e$
  - fields do not reach the wall at $z = \pm L$ $\quad E_z = \delta(z)\, e^{-x/\delta_e}$
- $b \ll 1 \Rightarrow$ spreading in z and evanescence in x

$$2LE_z = e^{-x/\delta_e} - 2 + \frac{1}{1 - e^{i\pi(z/L + x/h)}} + \frac{1}{1 - e^{i\pi(-z/L + x/h)}} \qquad h = b\delta_e$$

  - fields reach the wall
  - short scale structures in z are ES and decay quickly in $x \sim h \sim (m_e/m_i)^{1/2} L$
    - could create hot electrons on radial scale $(m_e/m_i)^{1/2} v_e \sim \rho_s$
  - long scale structures in z ($k_z = 0$) are EM and decay more slowly in $x \sim \delta_e$
- because the fields reach the wall for $b \ll 1$, they will generate an rf-sheath with finite $\Delta \propto \Lambda$
  - the limit Λ = 0 is not self-consistent $\Rightarrow$ need general Λ expansion

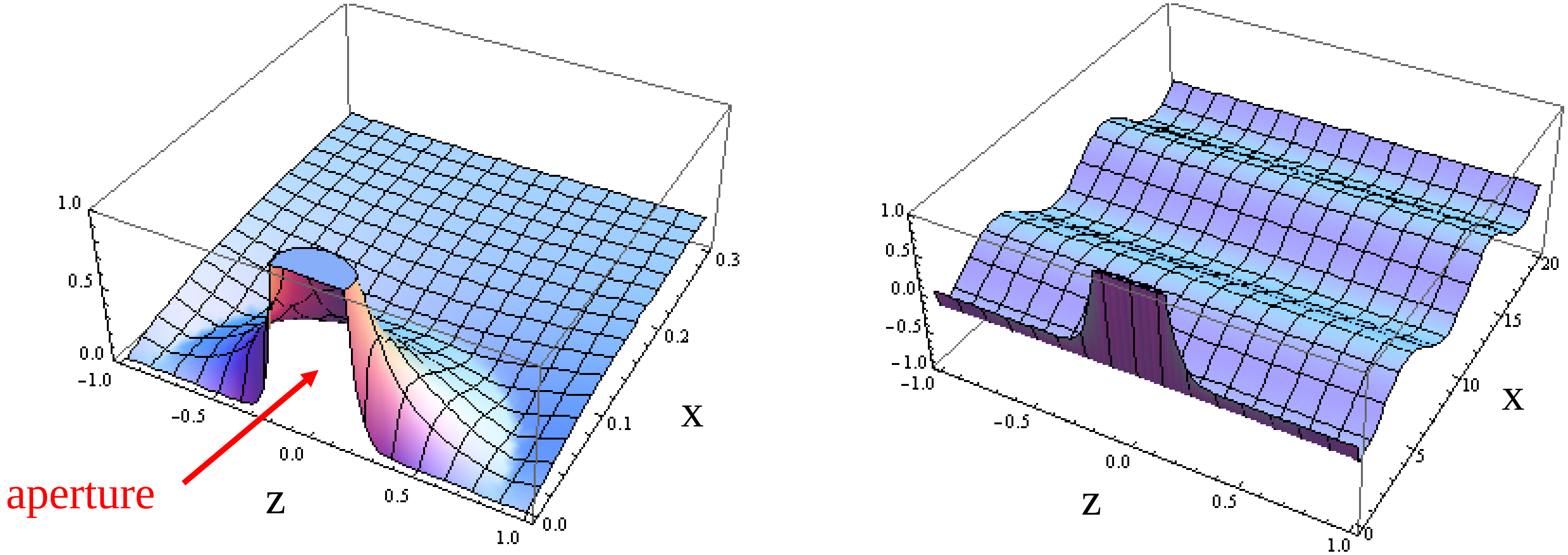
Solution for fixed finite Λ

- Gaussian source (aperture) $S(z)=\frac{1}{(2\pi)^{1/2}a}\exp\left(-\frac{z^2}{2a^2}\right)$
- $b \ll 1$ of most interest: so fields reach the wall $\Rightarrow$ $L < \delta_i$
- for $b \ll 1$, $\Lambda > 1$, imaginary root approaches $\eta = ib\left(\frac{\Lambda}{\Lambda-1}\right)^{1/2}$
- associate this root with the Alfvén mode from $\Lambda \to \infty$ limit

$$\eta^2+b^2=0 \qquad \Leftrightarrow \qquad k_z^2 v_a^2=\frac{\omega^2}{1-(\omega/\Omega_i)^2}$$

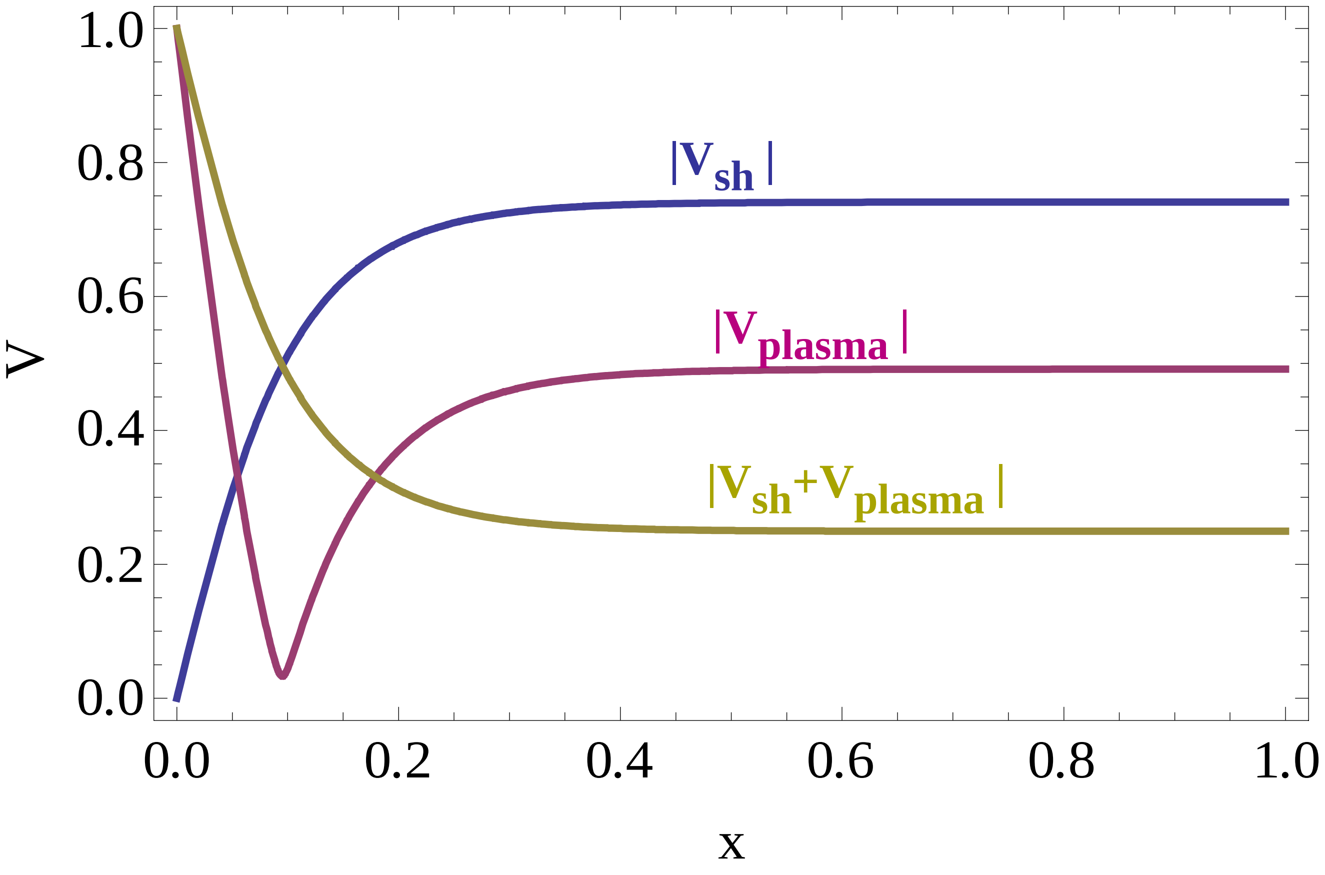
- Alfvén resonance normally occurs for real k_z and $\omega < \Omega_i$. Here, $\omega > \Omega_i$ but imaginary k_z allowed by sheath BCs

Field pattern and emergence of SPW for specified Λ



- left: |$E_Z(x, z)$| for b = 0.1, a = 0.1 and specified value of Λ = 3.
- right: Re[$E_Z(x, z)$] for the same case.
- note the appearance of asymptotic fields in x
  - radially propagating mode = Alfvén sheath plasma wave (SPW)
  - mode follows the sheath boundary radially into the plasma

A substantial fraction of the source voltage ends up on the sheaths



- total V not conserved because of EM effects
- note that V_{sh} ~ const for large x

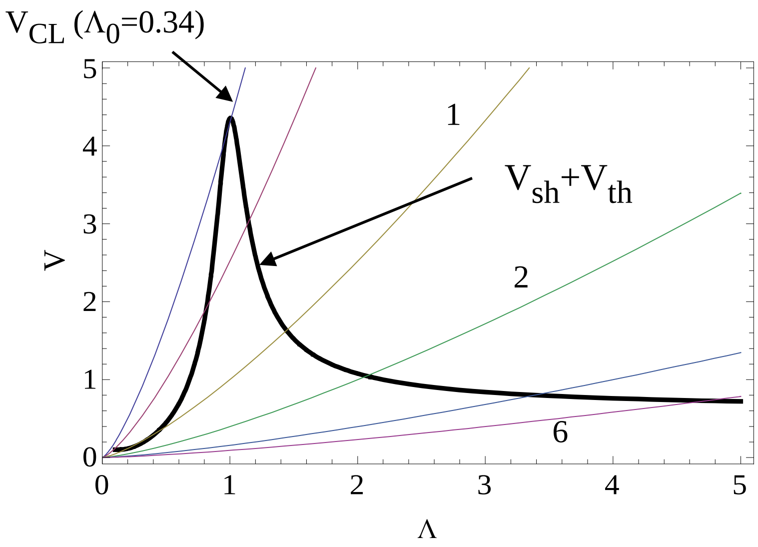
Self-consistent solution for Λ, Δ, and V_{sh} can have multiple roots

- use Child-Langmuir law: make Δ consistent with fields at wall

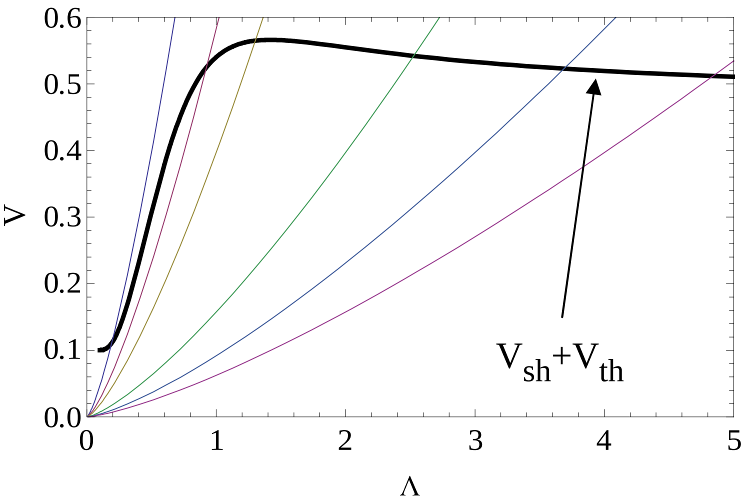
$$\Delta = \lambda_{de}\left|\frac{\alpha e V_{sh}}{T}\right|^{3/4} \qquad V_{sh} = \Delta\varepsilon_{\|}E_z(z = L)$$

comes from matching $\varepsilon_{\|}E_z$ across sheath-plasma interface

- graphical roots of $V_{sh} + V_{th} = \left(\frac{\Lambda}{\Lambda_0}\right)^{4/3} \equiv V_{CL}$

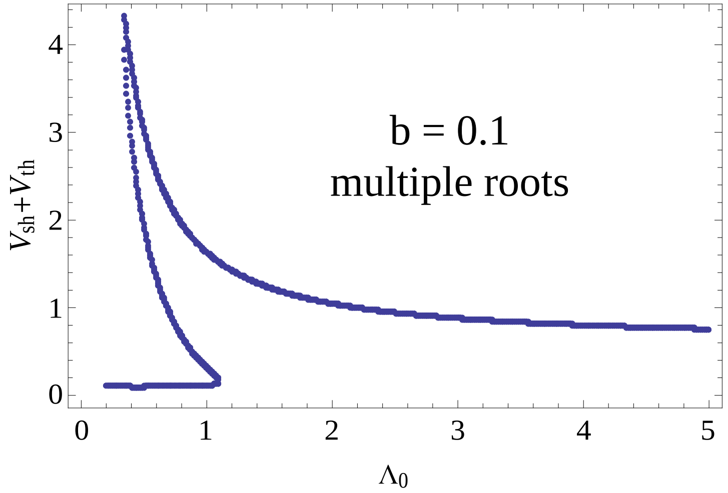


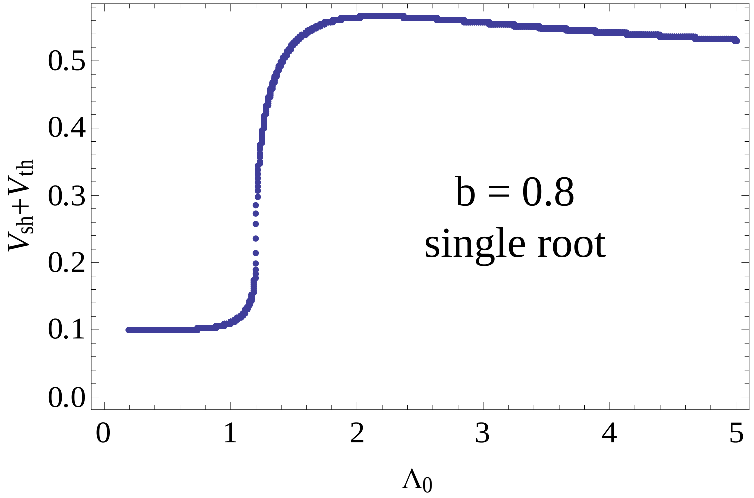
b = 0.1
multiple roots



b = 0.8
single root

Self-consistent sheath voltage (at x →∞) from SPW





- strong amplification possible for b << 1 in SPW resonant case ($\Lambda \sim 1$)
  - analogous effect seen in far-field “wave scattering” problem [D'Ippolito PoP 2008]
- as b increases to b ~ 1, V_{sh} decreases, resonant structure and multiple roots disappear
  - get critical Λ_0 at which sheath goes from thermal to rf-dominated

Summary

- SW fields emitted by a localized source propagate and evanesce into the SOL
- SW interaction with the wall, and concomitant rf-sheath formation is possible in some parameter regimes
  - tenuous plasmas $n_e < n_{lh}$ ($\omega > \omega_{lh}$) for which the SW propagates as resonance cone (RC) without spreading
  - dense plasmas $n_e > n_{lh}$ ($\omega < \omega_{lh}$) with nearby limiters, typically $L_{||} < \delta_i$
- In the dense plasma case studied here, the mechanism for wave-field coupling to the wall sheaths involves the sheath-plasma wave, for which we obtain a new electromagnetic dispersion relation related to the Alfvén wave.
- The SPW carries the SW fields and sheath voltages radially into the plasma along the surface of metal structures.
- A numerical treatment of this problem in realistic SOL geometry will be very interesting, and is in progress [H Kohno et al.]