Validity condition for the local sheath impedance boundary condition and a non-local generalization

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Abstract. ICRF sheaths can cause unwanted interactions of high-power RF waves with material surfaces in magnetic fusion devices. In previous work, a local RF sheath impedance boundary condition (BC) was derived for use in ICRF codes together with a microscale (i.e., Debye or sheath width scale) model for obtaining the sheath impedance used in that BC. This local RF sheath BC matches the normal component of current and electrostatic potential across the sheath-plasma interface. Collapsing the matching conditions at the sheath-plasma interface to a BC depends on the assumption of scale separation, which can be violated when conditions along the local radius of curvature of the surface vary sufficiently rapidly. The validity condition is explored in this contribution, with special attention to the case where the magnetic field approaches being tangent to the surface. When the local sheath BC no longer applies, a non-local sheath BC is developed under the assumption of a more relaxed scale separation assumption. It is shown that the non-local sheath BC reduces to the previous local sheath BC under appropriate conditions. A surface-integrated sheath admittance parameter [H. Kohno and J. R. Myra, this conference] describes the 2D physics in the new BC.

INTRODUCTION

It is widely accepted that under unfavorable circumstances ion cyclotron range of frequency (ICRF) sheaths can cause unwanted interactions of high-power RF waves with material surfaces in magnetic fusion devices. The physics and consequences of RF sheaths are reviewed in Ref. 1, experimental results and progress in modeling ICRF-edge plasma interactions are discussed in Refs. 2 - 4.

In previous work,[1,5] a local RF sheath impedance boundary condition (BC) was derived for use in ICRF codes together with a microscale (i.e., ion sound radius and Debye or sheath width scale Δ_m) model[6] for obtaining the sheath impedance used in that BC. This local RF sheath BC matches the normal component of RF current (including both particle and displacement currents) and electrostatic potential across the sheath-plasma interface.

The interaction of RF waves in the quasi-neutral plasma volume with this local sheath BC was studied in a series of papers using the rfSOL code.[7,8] In the limit of asymptotically large RF sheath voltages the local RF sheath BC usually reduces to a quasi-insulating limit,[1,8] often referred to as the "wide sheath" limit[9] which has been explored in Europe using the SSWICH code.[3,4,9,10] In the US, in addition to rfSOL, the local RF sheath BC has been implemented in several other modeling codes including Petra-M,[11] Vorpal,[12] Stix[13] and COMSOL.[14]

All of these models are based on a fundamental assumption: collapsing the matching conditions at the sheath-plasma interface to a BC depends on the assumption of scale separation, $\Delta_m << \lambda_{rf}$, where λ_{rf} is the scale of the RF waves and of surface variations. Scale separation can be violated when surface variations have radii of curvature $R_c \sim \Delta_m$ as is currently being studied in a two-dimensional microscale sheath model.[15] Additionally, we show in this contribution that the validity condition for the local sheath BC breaks down for much larger R_c near points where the magnetic field is tangent to the surface.

The properties of sheaths on curved surfaces also arises in the interpretation of Langmuir probe data. A body of work in this area, mostly for static sheaths, has considered, in particular, modeling the effective collecting area of probes in magnetized plasmas since this area may not correspond to the geometrical probe area due to ion finite Larmor radius effects. The reader is referred to Refs. 16 -18 and references therein. In high voltage RF sheaths, the thermal ion Larmor radius tends to be less important than the Larmor radius based on T_e through the sound speed and the one based on the RF potential energy eV_{rf}; this physics is captured by the fluid modeling cited previously.

Near magnetic tangency points, when the local BC no longer applies, a non-local sheath BC is developed in the following under the assumption of a more relaxed scale separation assumption. It is shown that the non-local sheath BC reduces to the previous local sheath BC under appropriate conditions. The results from a 2D microscale sheath model[15] may then enter the new boundary condition in the form of a surface-integrated admittance. In this paper only isolated tangency points on a gently curved surface are considered; we do not treat higher-dimensionality situations where the magnetic field is tangent to a surface along a line or an area. It remains to be seen whether the latter situations are realistic in practice given surface irregularities and magnetic curvature.

THE LOCAL SHEATH BC AND ITS VALIDITY

The local RF sheath BC takes the form

$$\mathbf{E}_t = \nabla_t (z_{sh} J_n) \tag{1}$$

where $\mathbf{E}_t$ and $J_n = \mathbf{J}\cdot\mathbf{n}$ represent tangential and normal RF field quantities on the plasma side of the sheath-plasma interface, $\mathbf{n}$ is the unit vector normal to the surface pointing into the plasma and the sheath impedance parameter z_{sh} is obtained from a 1D microscale sheath model. Scale separation allows the macroscale RF wave problem to be solved for $\mathbf{E}_t$ and J_n knowing only z_{sh} at the interface. Scale separation also allows the microscale sheath problem to be solved for z_{sh} in a 1D model knowing only values of the sheath potential Φ_{sh} or J_n at the interface, i.e., the RF current or voltage driving the sheath, or some combination of them.

In the following discussion we will frequently use the term 'sheath surface' to mean the sheath-plasma interface. On the microscale this surface corresponds to the plasma-facing boundary of the magnetic presheath, since the impedance z_{sh} measures the voltage drop between that location and the wall. On the macroscale, the sheath surface is for all practical purposes just the material surface.

If the tangential gradient along the sheath surface, or the normal component of the current J_n varies rapidly along the sheath surface, on a scale comparable to the sheath width, the 1D sheath model, and indeed the entire concept of a *local* BC no longer apply. Here the total sheath width is roughly $\Delta_m \sim \Delta + \rho_s$ where $\Delta \sim \lambda_{de}(eV_{rf}/T_e)^{3/4}$ is the width of a Child-Langmuir non-neutral sheath and $\rho_s \sim c_s/\Omega_i$ is the magnetic presheath width. The 1D condition can be violated if the sheath surface is sharply curved, with radius of curvature of order Δ_m. It can also be violated for much larger radii of curvature near magnetic tangency points, which is the primary focus of this paper. To see this, consider the incident wave current

$$J_n = -i\omega D_n = -i\omega\varepsilon_0 \mathbf{n}\cdot\vec{\vec{\varepsilon}}\cdot\mathbf{E} \approx -i\omega\varepsilon_0 b_n \varepsilon_{\parallel} E_{\parallel} \tag{2}$$

where in the last step we use $b_n = \mathbf{b}\cdot\mathbf{n}$, $E_{\parallel} = \mathbf{E}\cdot\mathbf{b}$ and approximate

$$\vec{\vec{\varepsilon}} \approx \mathbf{bb}\varepsilon_{\parallel} + O(\varepsilon_{\perp})\,. \tag{3}$$

At a magnetic tangency point where $b_n = 0$, J_n is reduced by a factor of order $\varepsilon_{\perp}E_{\perp}/E_{\parallel}\varepsilon_{\parallel} \sim \varepsilon_{\perp}k_{\perp}/k_{\parallel}\varepsilon_{\parallel}$ compared to its value when $b_n \sim 1$. Here the estimate is made for an electrostatic slow wave. As an example, for a case with density well above the lower hybrid resonance density one obtains the reduction factor $\varepsilon_{\perp}k_{\perp}/k_{\parallel}\varepsilon_{\parallel} \sim (m_e/m_i)^{1/2}$. The inclusion of fast waves and electromagnetic effects may change this rough estimate; however, it is sufficient to illustrate the source of the difficulty. For a magnetic tangency point on a surface of radius of curvature R_c, we have $b_n = 0$ at the tangency point and for the example $b_n \sim (m_e/m_i)^{1/2}$ at a distance $L_c \sim R_c\ (m_e/m_i)^{1/2}$ from the tangency point as measured along the surface. L_c is the characteristic parallel scale length of J_n over which it changes by order unity. If $L_c \sim \Delta_m$ or less, the local sheath BC model is violated because the scales parallel and perpendicular to the wall (and to $\mathbf{B}$) are comparable. See Fig. 1. For this example, $b_n \sim (m_e/m_i)^{1/2} \sim \theta_c$ where θ_c is the critical angle depicted in the figure.

Including densities near or below lower hybrid resonance, a critical radius of curvature may be estimated as $R_{crit} \sim \Delta_m(\varepsilon_{\parallel}/\varepsilon_{\perp})^{1/2}$ such that the local sheath model is expected to be valid only when $R_c > R_{crit}$. Figure 2 presents R_{crit} as a function of density for some illustrative tokamak edge parameters. It is seen that a wide range of values are possible, and also that R_{crit} becomes very large near the lower hybrid resonance density. If instead of considering the condition arising from the incident wave current, one considers the total current arising from both the incident and sheath-reflected waves, it can be shown that the sheath impedance enters the necessary criterion for validity, and that R_{crit}/Δ_m is again typically much larger than unity. These estimates are rough a-priori estimates of necessary (but not obviously sufficient) conditions for validity of the local sheath model; better estimates will have to await a numerical comparison of the local sheath model with the implementation of more advanced models such as the non-local model developed here. Also, some effects not treated in the previous 1D microscale sheath model[5,6] may be present even when $R_c > R_{crit}$.

It is not yet fully explored how violation of the local BC at isolated points affects quantities of important practical interest, such as the flux of impurity sputtering or RF power deposition. However, previous investigations found that rapid and likely unphysical fine scale oscillations occur near tangency points.[19] The goal of this contribution is to seek a physical means of generalizing the local RF sheath BC to avoid such problems.

When the local model no longer applies, in general one must solve a 2D or 3D problem in which there is no scale separation between the RF wavelength and the sheath width, $\Delta_m \sim \lambda_{rf}$. The nonlinear, non-neutral sheath physics must be treated in the same domain and with the same (unified) model that solves for the global electromagnetic RF wavefields. This may be impractical, and motivates the search for an intermediate ordering in which the microscale and macroscale problems may still be separated, by giving up on the ideal of a local BC. In this situation, RF currents can flow along the surface coupling different points on the sheath surface, hence the non-locality.

Let τ represent distance measured along the sheath surface and λ_τ represent the scale length along the sheath surface over which conditions vary. (For the geometry shown in Fig.1, $\lambda_\tau \sim L_c$ but we keep the notation more general.) The idea investigated here assumes that $\lambda_\tau \sim \Delta_m$ but the RF waves of importance still satisfy $\Delta_m << \lambda_{rf}$. This could be the case if, for example, very short wavelength RF modes are strongly damped. In this situation, scale separation still permits the concept of a BC: one that is local on the scale λ_{rf} but global on the scale of Δ_m. Formally we consider an intermediate scale length σ along the surface satisfying

$$\Delta_m \sim \lambda_\tau << \sigma << \lambda_{rf} \quad (4)$$

Then we match the *total* sheath current and plasma current along a segment of surface of order σ as opposed to matching the *local* sheath and plasma current density at every point. As will be seen, this results in an integral sheath BC.

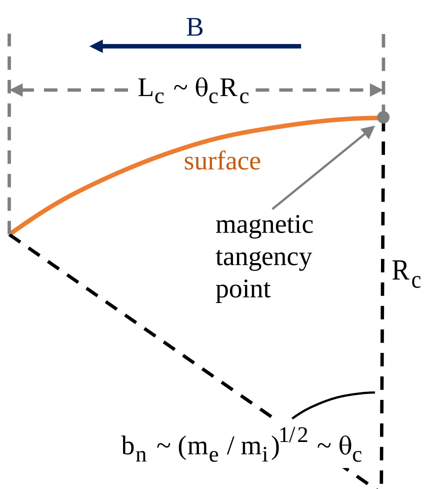

FIGURE 1. Geometry of a magnetic tangency point on a surface of radius of curvature R_c for the example case discussed in the text.

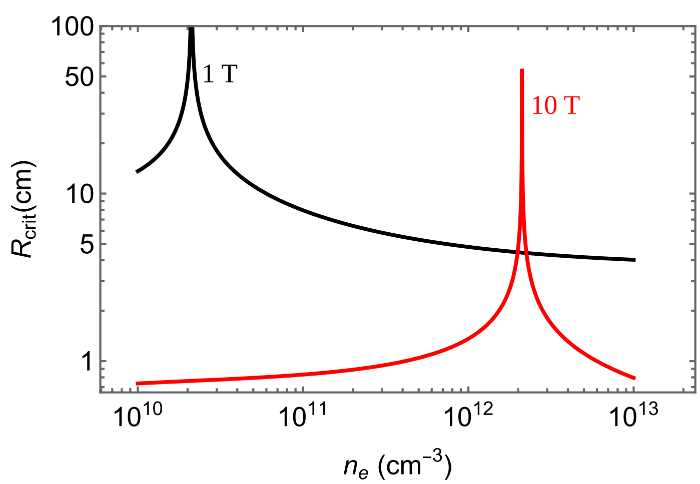

FIGURE 2. The critical radii of curvature R_{crit} for B = 1 T and 10 T, using deuterium, $\omega = 3\Omega_i$, V_{rf} = 200 V and T_e = 20 eV.

DERIVATION OF THE INTEGRAL SHEATH BC

For simplicity of presentation, we consider a 2D problem, homogeneous in a third dimension, so that the sheath surface is a line in the 2D plane of the problem. The results are easily generalized to 3D. The total plasma current (i.e., current from the plasma side of the interface) along a segment of surface is then

$$I_{pl} = \int_{\sigma} d\tau' J_{n,pl} \tag{5}$$

where the integration is over a region of order σ and integration over a unit length in the third dimension is implied, so that I_{pl} has units of current. Note that the σ integration smooths out the small scales of order λ_τ where the problematic variations discussed in Eqs. (2) and (3) arise.

On the sheath side, the sheath current is formally

$$I_{sh} = \int_{\sigma} d\tau' J_{n,sh} \equiv -Y_{sh}\Phi_{sh} \ . \tag{6}$$

The final part of Eq. (6) defines a 'global' sheath admittance,[15] i.e., global for the microscale problem. (The minus sign arises because the sheath admittance Y_{sh} is defined to be positive when in-phase current flows from the potential at the sheath entrance Φ_{sh} to the grounded wall. Recall that **n** is the unit normal pointing from the surface *into* the plasma.) Y_{sh} (uppercase Y) is dimensionally a true admittance (in SI units of Siemens). The local admittance parameter y_{sh} or y_{sh}^{1D} (lowercase y), introduced in the 1D microscale modeling[5] and used here subsequently, is correctly named a specific admittance given in units of S/m^2. Matching $I_{pl} = I_{sh}$ and $\Phi_{pl} = \Phi_{sh}$ results in

$$-\Phi_{pl} = Z_{sh} \int_{\sigma} d\tau' J_{n,pl} \tag{7}$$

where the global sheath impedance is $Z_{sh} = 1/Y_{sh}$. Taking the tangential gradient of Eq. (7) assuming the that sheath field is electrostatic, ($\Delta_m << c/\omega$)

$$\mathbf{E}_t = \nabla_t \left(Z_{sh} \int_{\sigma} d\tau' J_{n,pl} \right). \tag{8}$$

This is the desired integral sheath BC once we make the definitions of the integration and of Z_{sh} or Y_{sh} more precise. Note that the present model is constructed to handle tangency points on a smooth but curved surface; therefore, although J_n can vary rapidly for reasons already explained, E_t itself is by construction guaranteed to be smooth on the scale of σ. Eq. (8) is rewritten as

$$\mathbf{E}_t(\tau) = -\nabla_t \left(\frac{\Phi_{sh}(\tau) \int_{\sigma(\tau)} d\tau' J_{n,pl}(\tau')}{\int_{\sigma(\tau)} d\tau' J_{n,sh}(\tau')} \right) \to -\nabla_t \left(\frac{\Phi_{sh} \int d\tau' \, w(\tau,\tau') J_{n,pl}(\tau')}{\int d\tau' \, w(\tau,\tau') J_{n,sh}(\tau')} \right) \tag{9}$$

where w is a weight-function kernel, with scale length σ, for example

$$w(\tau,\tau') = e^{-(\tau-\tau')^2/(2\sigma^2)}. \tag{10}$$

For numerical implementation, other choices for $w(\tau, \tau')$ may be more convenient, for example the 'boxcar' function $w(\tau, \tau') = H(\tau-\tau'+\sigma)-H(\tau-\tau'-\sigma)$ where H is the Heaviside step function.

To evaluate the denominator, consider a localized sheath variation on the scale λ_τ embedded in a larger region of order σ. In the vicinity of strong surface variation, taken to be near $\tau = 0$ for this part of the calculation, a 2D solution of the microscale sheath problem is required. Far away from $\tau = 0$ the 1D sheath model is adequate.

$$J_{n,sh} = J_{n,sh}^{2D} = \delta J_{n,sh}^{2D} + J_{n,sh}^{1D} \tag{11}$$

where

$$\delta J_{n,sh}^{2D} = J_{n,sh}^{2D} - J_{n,sh}^{1D} \tag{12}$$

and $\delta J_{n,sh}^{2D}$ is only significant near $\tau' = 0$. Substituting Eq. (11) into Eq. (9) and dividing numerator and denominator of Eq. (9) by Φ_{sh} yields

$$\mathbf{E}_t(\tau) = \nabla_t\left(\frac{\int d\tau'\, w(\tau,\tau')J_{n,pl}(\tau')}{\delta Y_{sh} e^{-\tau^2/2\sigma^2} + \int d\tau'\, w(\tau,\tau')y_{sh}^{1D}(\tau')}\right) \tag{13}$$

where

$$\delta Y_{sh} = -\frac{1}{\Phi_{sh}}\int d\tau'\, \delta J_{n,sh}^{2D}(\tau') \tag{14}$$

$$w(\tau,0) = e^{-\tau^2/(2\sigma^2)} \tag{15}$$

$$\frac{1}{\Phi_{sh}(\tau)}\int d\tau'\, w(\tau,\tau')J_{n,sh}^{1D}(\tau') \approx \int d\tau'\, w(\tau,\tau')\frac{J_{n,sh}^{1D}(\tau')}{\Phi_{sh}(\tau')} = -\int d\tau'\, w(\tau,\tau')y_{sh}^{1D}(\tau')\,. \tag{16}$$

In obtaining these expressions use has been made of the ordering in Eq. (4). Specifically, in the δY_{sh} integral, $w(\tau,\tau')$ may be evaluated at $\tau' = 0$ for the reasons stated above, i.e. that $\lambda_\tau << \sigma$. In the y_{sh}^{1D} term we can equate $\Phi_{sh}(\tau) \approx \Phi_{sh}(\tau')$ inside the integral because $\Phi_{sh} = \Phi_{pl}$ is supposed to vary on a scale λ_{rf} that is large compared with σ.

CONCLUSION

Eq. (13) is the integral sheath BC. Generalizing slightly to account for multiple tangency points at locations τ_k along the sheath surface (as opposed to just one such point at $\tau = 0$ in the previous section) we may write the BC as

$$\mathbf{E}_t = \nabla_t\left(Z_{sh} I_{pl}\right) \tag{17}$$

where $\mathbf{E}_t$ and I_{pl} are evaluated on the plasma side of the sheath-plasma interface, and where

$$I_{pl}(\tau) = \int d\tau'\, w(\tau,\tau')J_{n,pl}(\tau') \tag{18}$$

$$\frac{1}{Z_{sh}(\tau)} = \sum_k \delta Y_{sh,k} w(\tau,\tau_k) + \int d\tau'\, w(\tau,\tau')y_{sh}^{1D}(\tau')\,. \tag{19}$$

Here, τ_k are the tangency points, i.e., the points where the 2D sheath theory should be applied. In Eq. (19), $\delta Y_{sh,k}$ is given by Eq. (14) together with the solution of the global admittance from the 2D sheath problem,[15] and $\delta Y_{sh,k}$ is independent of τ.

In the case where there are no tangency points, we may take σ to be small compared with λ_τ which would then be of order λ_{rf}, i.e., $\Delta_m << \sigma << \lambda_\tau \sim \lambda_{rf}$. Then $w(\tau,\tau')$ is proportional to a δ-function. For Eq. (10), $w(\tau,\tau') \rightarrow \sigma(2\pi)^{1/2}\delta(\tau-\tau')$, we may drop the $\delta Y_{sh,k}$ term and the integral BC reduces to the previous local sheath BC, Eq. (1), with $z_{sh} = 1/y_{sh}^{1D}$.

In many situations, the scale lengths Δ_m and λ_{rf} may be disparate giving some latitude in the choice of σ. In practice, it is likely that the scale of σ would be chosen as slightly larger than the smallest scale length that can be resolved in the RF code. Then the integrals appearing in the BC would involve only a few nearest neighbor cells in the macroscale code. In effect, the proposed new BC smooths the local current density thereby eliminating problematic tangency points. It also adds a correction, $\delta Y_{sh,k}$, that arises from RF sheath effects that are intrinsically 2D. It may be possible that the 2D effects from $\delta Y_{sh,k}$ could be evaluated and tabulated for a few generic geometric situations of interest, such as tangency point interactions for various radii of curvature or interactions near the corner of a limiter.

The assumption that the scale of the RF wavefields, λ_{rf} can be large compared with the scale of surface variations λ_τ, is both the weakness and strength of the proposed method. It is likely that wavelengths of order λ_{rf} incident on the surface will indeed generate some wave response on the λ_τ scale. The approach relies on the assumption, to be tested, that the λ_τ scale waves so generated will not significantly affect the self-consistent global wave solution. This amounts to an insensitivity of the solution to the parameter σ.

Thus, although the present approach makes some asymptotic orderings that may sometimes be difficult to satisfy in practice, it is hoped that even in marginal cases the approach will provide a practical and physically motivated, if not fully rigorous, means of dealing with tangency points in the sheath BC for RF simulations.

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