Turbulent transport regimes and the SOL heat flux width

J. R. Myra, D. A. D'Ippolito and D. A. Russell,
Lodestar Research Corporation, Boulder CO

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Introduction

- Need to understand the responsible mechanisms and resulting scaling of the SOL heat flux width λ_q.
  - Critical for viable operation of future machines, e.g. λ_q in ITER
- Simulation and theory using reduced edge/SOL turbulence models[1-5] show some agreement (λ_q magnitude and scaling) with experiment.
  - Overarching qualitative conceptual understanding of regimes and results is still lacking $\Leftarrow$ **the main goal of this work**
- The heuristic drift model[6] (associated with neoclassical effects[7]) has also been very successful in explaining experimental observations.
  - Here we also speculate on the relationship between the SOL turbulence and drift-based mechanisms.

[1] D.A. Russell et al., Phys. Plasmas **19**, 082311 (2012); and this meeting.
[2] J.R. Myra et al., Phys. Plasmas **18**, 012305 (2011).
[3] F. D. Halpern et al., Nucl. Fusion **53**, 122001 (2013) .
[4] F. Militello et al., Plasma Phys. Control. Fusion **55**, 074010 (2013).
[5] J. W. Connor et al., Nucl. Fusion **39**, 169 (1999).
[6] R.J. Goldston, Nucl. Fusion **52**, 013009 (2012).
[7] C.S. Chang et al., FY-2010 JRT and follow-on work (unpublished)

Turbulence provides a mechanism for sustaining the SOL width λ_q

- Heat flux balance in the SOL
  - $\nabla_{||} q_{||} = -\nabla_{\perp} \cdot q_{\perp}$
  - $q_{\perp} \sim D_{turb}\, p/\lambda_p$ where $D_{turb} \sim \gamma \langle \tilde{v}_x^2 \rangle / |\omega|^2$
  - perpendicular scale length of pressure is λ_p
    - $1/\lambda_p$ drives the turbulence
  - parallel heat flux $q_{||} = g\, nTc_s$
    - g is regime-dependent factor
  - parallel scale is $L_{||} = qR$
- Instabilities of interest
  - **curvature** driven (ideal, resistive, resistive X-pt = RX)
  - Kelvin-Helmholtz (**KH**)
  - collisional drift-wave (**DW**)
  - others?

$$\lambda_q = \frac{qR\gamma \langle \tilde{v}_x^2 \rangle}{gc_s |\omega|^2 \lambda_p}$$

need specific inputs for γ, saturation …

Fluctuation amplitude $\tilde{v}_x$ is determined by saturation: several regimes are possible

- **Wave-breaking** $\dfrac{k_x \tilde{v}_{Ex}}{\omega} \sim 1$
  - equivalent to equating the perturbed and equilibrium pressure gradients (pressure-convective saturation)
- Shear flow generation from **Reynolds** stress

$$\gamma = v'_{Ey} \equiv \frac{v_{Ey}}{\lambda_E} = \frac{k_x k_y}{\nu} \frac{\left\langle \tilde{\Phi}^2 \right\rangle}{\lambda_E^2}$$

  - here λ_E = scale length of the radial electric field
  - and ν = zonal flow dissipation rate
  - beats wave-breaking when $k_x k_y \lambda_E^2 \nu < \gamma$ or for global modes when $\nu < \gamma$
- **Mean flow** suppression (not really a saturation mechanism)
  - an important case is H-mode where we estimate that E×B and diamagnetic flows balance

$$\gamma = v'_{di} \equiv \frac{c_s \rho_s}{\lambda_p^2}$$

Relevant wave-number estimates depend on the regime and type of instability

- **Quasi-local** limit $k_y\lambda_p >> 1$
  - Conventional resistive modes typically require a high $k_y \sim k_\eta \equiv \left(\frac{\Omega_e}{\nu_e}\right)^{1/2} \frac{\lambda_p^{1/4}}{qR^{3/4}\rho_s^{1/2}}$ from $\omega_\eta \gamma_{mhd} \sim \omega_a^2$
  - FLR-mitigated ballooning mode spectrum can peak where $\gamma \sim \omega_{*i} = k_y c_s \rho_s / \lambda_p$
  - For quasi-local modes, estimate $k_x^2 \sim k_y \lambda_p$
    - obtained from parabolic expansion of a generic eigenmode equation about the point of maximum growth.
- **Non-local** (global) modes $k\lambda_p \sim 1$
  - For these modes the radial eigenfunction overlaps the bulk of the driving gradient
  - Estimate $k_x \sim k_y \sim 1/\lambda_p$
- **Barrier limited** non-local modes $k_x \sim \pi / L_x$
  - Rapid changes in geometry or plasma profiles near the separatrix can radially confine low k_y modes to a smaller scale than they would otherwise have.
  - e.g.: electron adiabaticity barrier limits extent of interchange modes; sheaths may do the same: RX modes are limited by the width of the X-pt shear layer

The instability drive may, or may not, be local to the SOL

- "**Compact**" modes
  - When the driving gradients are in the SOL we estimate

$$\lambda_p \sim \lambda_q$$

  - This is frequently the case for quasi-local instabilities
- "**Distributed**" modes
  - When the driving gradients are in the edge pedestal but large scale convective motions cause this turbulence to govern the SOL width[1,2] then λ_p and λ_q are independent parameters.
  - Here we regard λ_p = driving gradient = an *input*
    - later we discuss a rough estimate for λ_p in an H-mode.
  - λ_q = responding gradient = an *output*
  - Note that this distributed mode paradigm connects the pedestal properties to the SOL heat flux width.

Combining these leads to many possible scalings for λ_q

- 3+ types of instabilities
  - curvature-driven ideal or RX (a low k version of resistive), resistive, FLR
  - DW
  - KH
- 3 different eigenfunction regimes
  - Q = quasi-local
  - N = non-local
  - B = barrier-limited non-local
- 3 different saturation/mitigation regimes
  - W = wave-breaking
  - R = Reynold's driven flows
  - M = mean flows
- 2 types of transport
  - C = compact (normally associated with Q)
  - D = distributed (normally associated with N or B)
- Some combinations are more physically interesting: concentrate on what we have seen in past and ongoing SOLT simulations.

An example: ideal curvature modes in the BWD case

- BWD = barrier-limited, wave-breaking, distributed
- same estimates apply for RX[8] modes
- Starting from λ_q on p. 3,
  - first use $\tilde{v}_x / \omega = 1/k_x = L_x/\pi$
  - then use $\gamma = c_s/(R\lambda_p)^{1/2}$
  - to get

$$\lambda_q = \frac{q}{g}\frac{R^{1/2}L_x^2}{\pi^2\lambda_p^{3/2}}$$

  - for compact modes we would set $\lambda_p = \lambda_q$ and solve for λ_q

[8] RX mode are moderate $k_y << k_\eta$ ideal modes in the OM but become resistive (disconnecting from good curvature) near the X-pts due to strong magnetic shear. The growth rate is of order of ideal MHD [J.R. Myra et al., Phys. Plasmas 7, 4622 (2000)]

Turbulent suppression in H-mode

- This case is QMC in our keyed notation

$$\frac{c_s}{(R\lambda_p)^{1/2}} = \gamma \le v'_{di} \equiv \frac{c_s \rho_s}{\lambda_p^2}$$

$$\Rightarrow \qquad \lambda_p \le R^{1/3} \rho_s^{2/3}$$

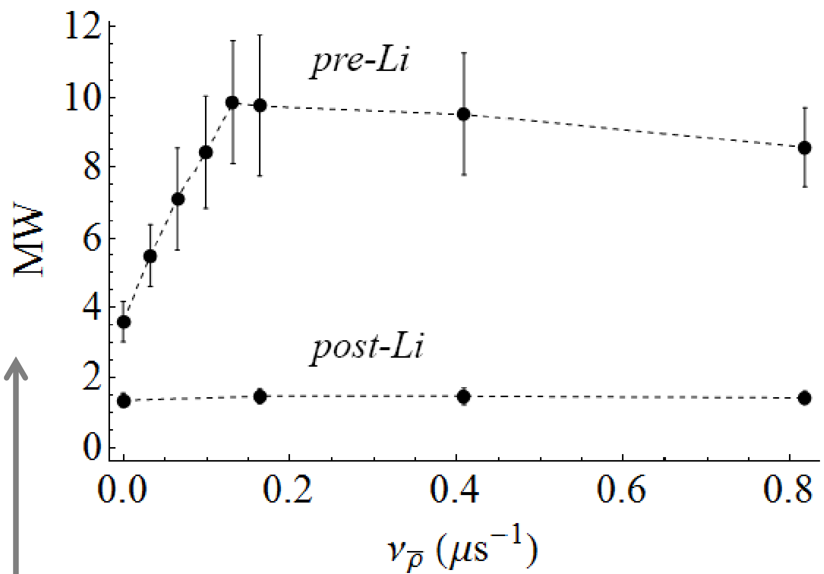
  - It is the condition for mean flows to stabilize interchange-like modes
  - It provides a rough limit on the pressure gradient in an H-mode assuming $v_E = -v_{di}$ i.e. that net fluid flows are small.
  - It is order of magnitude correct for NSTX
- Even when this condition is satisfied, there can still be instabilities:
  - near the maximum logarithmic pressure gradient the E×B shear is zero (assuming $v_E = -v_{di}$)
  - low k non-local distributed modes (N or B and D type) can grow centered at this location and still control the SOL width
  - also curvature enhanced KH modes

Summary table of some interesting cases

simplified for $q = g = 5$ (e.g. sheath transmission) to highlight R and ρ_s scalings

Instability	Key	Scaling for λ_q	Remarks
resistive (k_η)	QWC	$2.5\left(\frac{\nu_e}{\Omega_e}\right)^{2/7} R^{5/7}\rho_s^{2/7}$	Halpern[3]
ideal or RX (ω_{*i})	QWC	$(R\rho_s)^{1/2}$	larger for low k: NWC $\lambda_q \rightarrow R$!
ideal or RX	BWC	$R^{1/5}\left(\frac{L_x}{\pi}\right)^{4/5}$	L mode?
drift	QWC	$R^{1/3}\rho_s^{2/3}$	maximal estimate
ideal or RX	_MC	$R^{1/3}\rho_s^{2/3}$	H-mode; mean flow suppression
ideal or RX	NWD	$(\lambda_p R)^{1/2}$	large! even larger if compact; low-k ideal modes destroy SOL
ideal, RX or KH	BRD	$\frac{\nu}{\Omega_i}\frac{RL_x^2}{\pi^2\lambda_p\rho_s}$	~ ν/λ_p like SOLT; independent of γ
ideal or RX	BWD	$\frac{R^{1/2}L_x^2}{\pi^2\lambda_p^{3/2}}$	upper limit (BRD < BWD) SOLT with flow damping?
KH	BWD	$0.2\frac{L_x^2}{\pi^2}\frac{R\rho_s}{\lambda_p^3}$	
Heuristic Drift		$q\rho_s$	Goldston model[6]; not instability-based ($q\rho_s$ is a simplified order of mag. version)

SOLT simulations[1] for NSTX H-mode suggest BRD, BWD scaling



damping rate of sheared flows →

$$P_{sep} = 2\pi R \lambda_q b_\theta q_{||} \propto \lambda_q$$

- linear increase with ν in R regime;

$$\lambda_q = \frac{q}{g}\frac{\nu}{\Omega_i}\frac{R L_x^2}{\lambda_p \rho_s}$$

- plateau in W regime

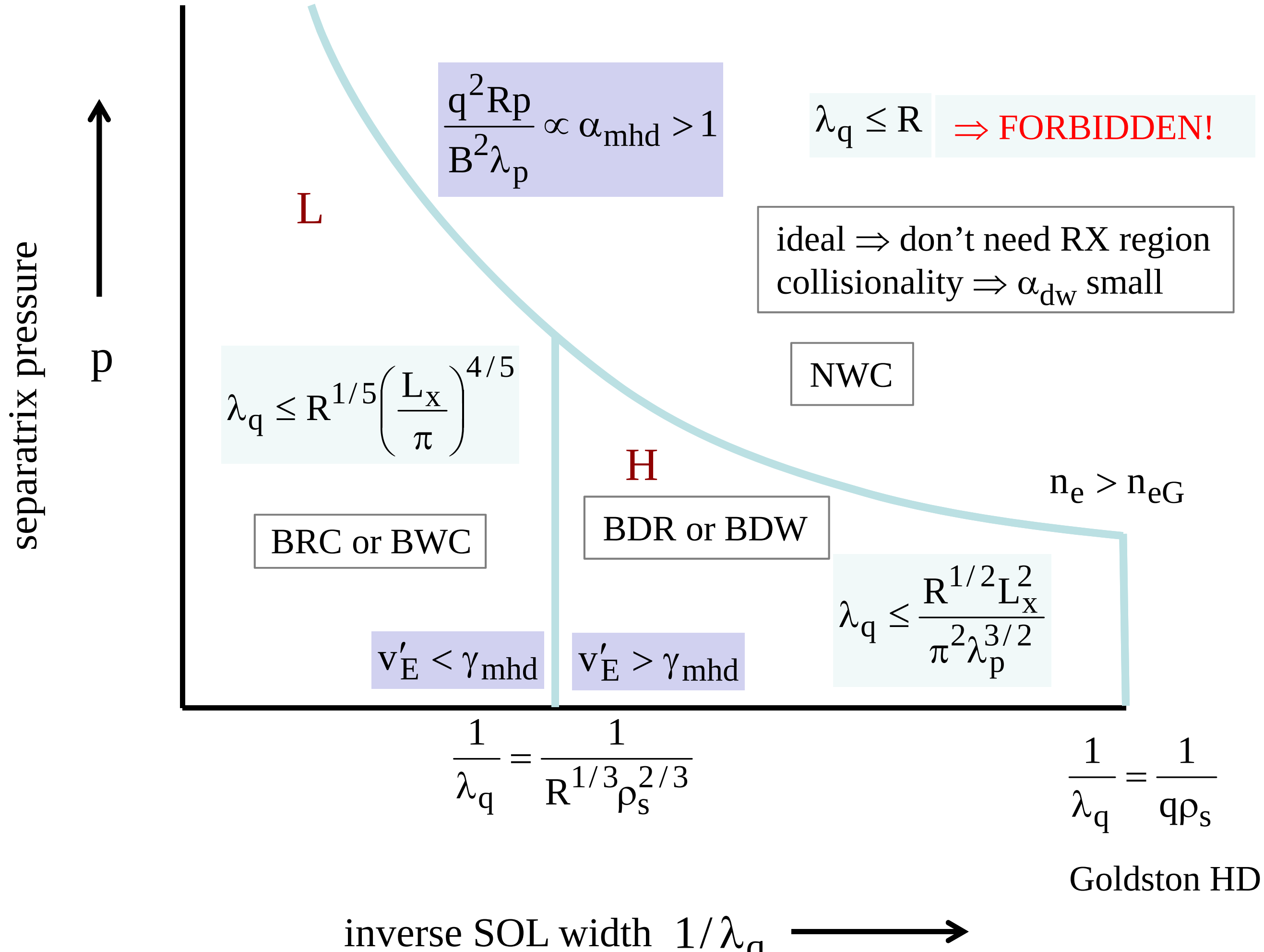
$$\lambda_q = f\,\frac{q}{g}\frac{R^{1/2} L_x^2}{\lambda_p^{3/2}}$$

- inverse scaling with λ_p consistent with pre-Li and post-Li [Russell talk]

Sample SOL width diagram

(with speculative connection to HD density limit)

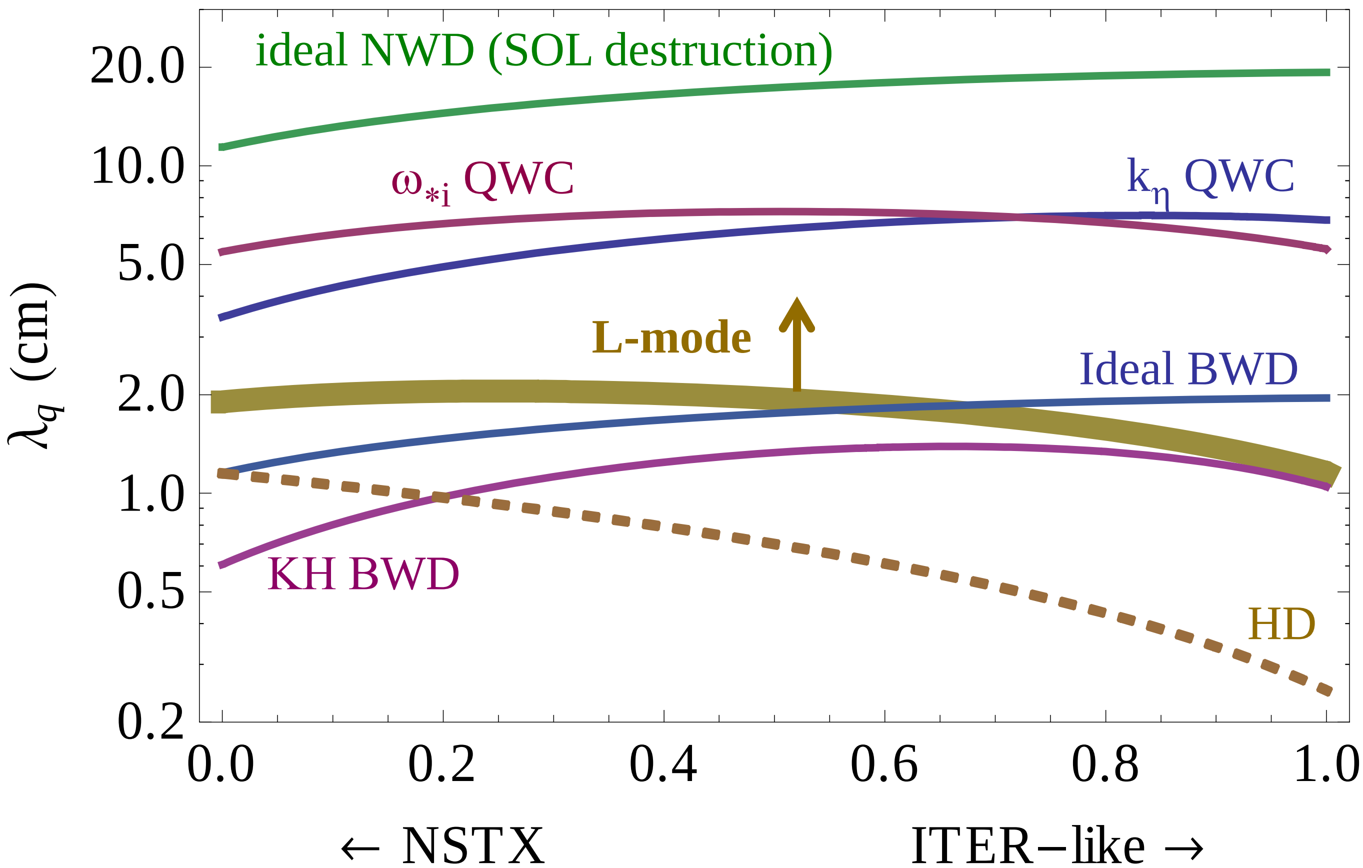
- a connection of heuristic drift (HD) model to density limit was proposed in
  [Goldston and Eich, 24th IAEA FEC, San Diego, October 8 - 13, 2012, paper IAEA-CN-197/TH/P4-19]



Notes on the diagram

- This is *not* a regime diagram for the L-H transition; it is mean to show how the predictions for λ_q change in the different regimes.
- L-mode scale lengths λ_p are long, and below the threshold for mean flow suppression. Compact modes are possible.
- The $R^{1/3}\rho^{2/3}$ boundary only applies to the L-mode side (compact). It gives a relatively wide SOL.
- In H-mode, not only is λ_p shorter, but λ_q is at a different location (distributed) so the resulting SOL width is much narrower than in L-mode, since $\lambda_p > \lambda_q$.
- Quoted estimates for λ_q are wave-breaking limit; Reynolds estimates will be smaller.
- When turbulence SOL widths exceed Goldston HD, could get a two-scale SOL; when Goldston HD width is larger, turbulence may be irrelevant.
- Approaching the α_{mhd} boundary in H-mode $\Rightarrow$ increased transport, broadened SOL which moves one up and along the curve to the L-mode regime.
- Strong perpendicular transport at the α_{mhd} boundary is consistent with parallel disconnection from the sheaths.

Order-of-magnitude estimates



- ad-hoc transition of parameters from NSTX (x = 0) to ITER-like (x = 1)
- wave-breaking estimate illustrated; Reynolds estimates will be smaller.
- turbulence results scale better than HD in going to ITER

Conclusions

- Simple hand-waving considerations for turbulent transport fluxes in various regimes can qualitatively explain some of the SOL width results seen in SOLT simulations: both scaling and order of magnitude.
- Detailed comparison with experiments remains, but present results do not seem unreasonable.
- The turbulent SOL heat flux width in L-mode and H-mode may depend on different transport mechanisms, i.e. separation of driving gradients (pedestal) and responding gradients (SOL) (i.e. compact vs. distributed)
- A speculative relationship is suggested between the turbulence and the heuristic drift mechanism for the SOL width, which may also relate to the density limit.
- Turbulence mechanisms tend to give λ_q a positive scaling with R. These are more favorable for large machines (like ITER) than the HD model which just depends on ρ_s.