Integrated Codes for ICRF-Edge Plasma Interactions*

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Introduction

● One of the outstanding practical problems in the use of ICRF antennas is predicting the magnitude and nature of local (and mainly nonlinear) ICRF-plasma interactions, which impact heating efficiency, impurity production, integrity of plasma-facing components, etc.

● RF-plasma interactions depend on the local density or plasma flux to the antenna and also modify them. This complicated interaction cannot be self-consistently modeled with present codes.

● The problem of computing the SOL density profile couples several areas of fusion physics:

- nonlinear rf-plasma interactions (rf sheath, ponderomotive, and PDI physics)
- nonlinear evolution of SOL turbulence (wave-breaking, blob formation and transport, etc.)
- atomic and surface physics: recycling, ionization, recombination, etc. (*not discussed here)*

● Lodestar and ORNL are developing an integrated suite of codes for

- computing self-consistent SOL density profile
- studying rf-turbulence interactions

● This work was initiated under a small-business collaborative research (STTR) grant which provided a unique opportunity to bring together various models and codes in rf and turbulence physics and develop an integrated suite of codes for quantitative modeling.

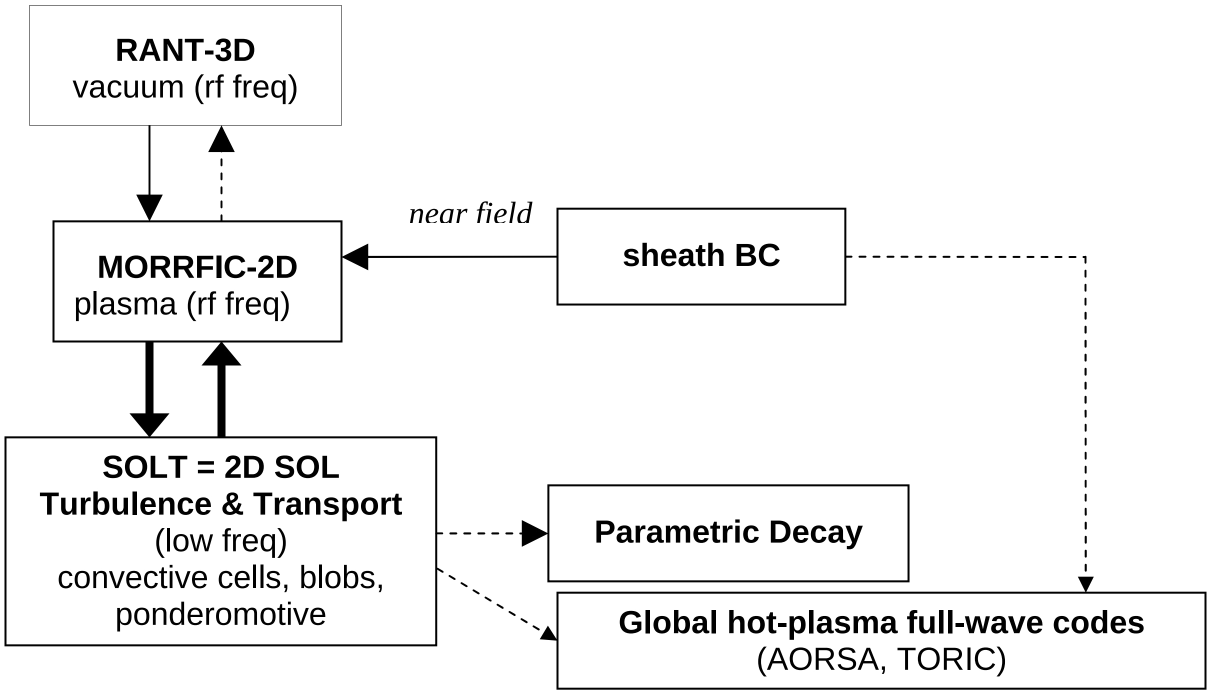
Outline of posters

1. Overview of the SOL physics problem
2. 2D MORRFIC antenna code development
3. RF Sheath BC for antenna codes
4. 2D SOLT turbulence and transport code development
5. Preliminary work on code integration
6. Future plans

Also see the adjacent poster by M. D. Carter

for more details on the MORRFIC code work

Schematic of Integrated Codes



Schematic diagram showing the interaction of the rf and edge physics codes. Heavy solid arrows indicate work already completed in Phase I of the STTR project; light solid arrows denotes work proposed in Phase II; dashed arrows indicates work to be done under other projects (e.g. the rf SciDAC project). In the Phase I "proof of principle" demonstration, the MORRFIC-transport code coupling was "open loop", as described in the text.

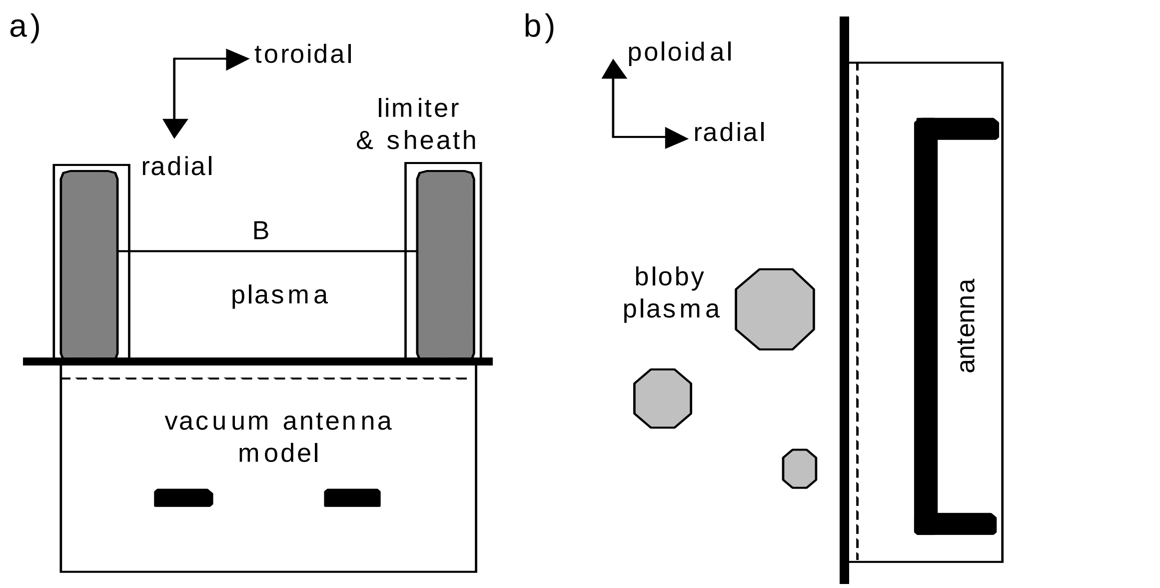
2D MORRFIC Antenna Code

MORRFIC is a 2D antenna code which can model plasma variations along field lines.

The following modifications were made to MORRFIC for this project:

- created a 2D model of the C-Mod antenna (with similar dimensions and loading to the RANT-3D model)
- incorporated vacuum sheath BCs
- two configurations:
  - "radial-toroidal" for sheath and ponderomotive studies

    $\Rightarrow$ sheath voltage $V_{sh}(x,y)$ and pond. potential $\Psi(x,y)$
  - "radial-poloidal" for studies of rf wave scattering by blobs



Simulation geometry for MORRFIC with plasma: a) 2D (radial-toroidal) simulations with plasma and limiters, b) 2D (radial-poloidal) simulations.

(see adjacent poster by M. Carter et al.
for initial MORRFIC code results)

RF Sheath BC

For rf field codes such as MORRFIC and AORSA, we have derived an **analytic rf sheath BC** (jump condition for the thin sheath layer with $\varepsilon_{zz} = 1 + i\nu$):

$$\nabla_t \cdot \mathbf{E}_t - \frac{\Delta}{1+i\nu} \nabla_t^2 D_z = 0 \quad , \quad B_z = 0 \quad .$$

This replaces the usual BCs in a conductor, $\mathbf{E}_t = 0$, $B_z = 0$. Here, the rf fields $\mathbf{E}_t$ and $D_z = \mathbf{e}_z \cdot \boldsymbol{\varepsilon} \cdot \mathbf{E}$ are defined on the *plasma* side of the sheath-plasma interface and the subscripts t $\Rightarrow$ *tangential* and z $\Rightarrow$ *normal* to this surface.

In the capacitive sheath limit, the dissipation parameter $\nu \sim \omega_{pi}/\omega << 1$ and the sheath voltage V_{sh} and sheath width Δ are given by

$$V_{sh} \equiv \int_{z=0}^{\Delta} dz\, E_z \approx \frac{D_z\, \Delta}{1+i\nu} , \qquad \Delta = \lambda_D \left(\frac{eV}{T_e} \right)^{3/4} ,$$

using the continuity of D-normal and the Child-Langmuir law. The system of equations is closed by the power relations

$$\nu = \frac{4\pi\Delta}{\omega V^2} \left(\frac{P_{sh}}{A} \right) , \quad \frac{P_{sh}}{A} \equiv Z n_e c_s T_e \xi\, h(\xi) \frac{I_1(\xi)}{I_0(\xi)} ,$$

where P_{sh} = sheath power dissipation, $\xi = ZeV/T_e$, and $h(\xi) = (0.5 + 0.3\,\xi)/(1 + \xi)$.

- **iterate these eqs. to convergence (or use root finding in 2 parameter space) to obtain the self-consistent solutions for the rf fields, sheath voltage V_{sh}, and sheath power dissipation P_{sh}**
- **no need for increased grid resolution near boundaries to resolve sheaths**
- **eliminates need for extensive post-processing to get sheath information!**

Effect of Insulating Limiters on BC

The sheath properties can be represented by an equivalent **lumped-parameter circuit** model with a resistance R_{sh} and a capacitive impedance $Z_{C,sh}$ given by

$$R_{sh} = \frac{V_{sh}^2}{2P_{sh}}, \qquad |Z_{C,sh}| = \frac{4\pi\Delta}{\omega A}$$

This model can be extended to include the resistance and capacitive impedance of a **thin insulating coating** applied to the surface beneath the sheath:

$$R_{in} = \frac{\eta d}{A}, \qquad |Z_{C,sh}| = \frac{4\pi d}{|\varepsilon_{in}|\omega A}$$

where η is the effective ac volume resistivity, d is the thickness of the insulator, ε_{in} is the dielectric constant of the insulator, and ω is the rf frequency.

The voltage drop across the sheath V_{sh} relative to the total voltage $V = V_{sh} + V_{in}$ is given by

$$\frac{V_{sh}}{V} = \frac{Z_{sh}}{Z_{sh} + Z_{in}},$$

where $1/Z_j = 1/R_j + 1/Z_{Cj}$ for j = sheath or insulator. Note that insulating limiters can reduce the sheath voltages: **$V_{sh}/V \to 0$ as $Z_{in}/Z_{sh} \to \infty$**.

Extending our rf sheath model to include two thin layers (insulator + sheath + plasma) gives the **same BC** on the rf fields in the plasma along with the following expression for the **voltage split**:

$$V_{sh} = -D_z \frac{\Delta z_{sh}}{\varepsilon_{zz,sh}},$$

$$V = V_{in} + V_{sh} = -D_z \left(\frac{\Delta z_{in}}{\varepsilon_{zz,in}} + \frac{\Delta z_{sh}}{\varepsilon_{zz,sh}} \right).$$

where $\varepsilon_{zz,sh} = 1 + i\,\nu$ and Δz = layer thickness.

Model problem with sheath BC

Consider an EPW propagating across the magnetic field ($\mathbf{B}_0 = B_0\, e_z$, $\mathbf{k} = k\, e_x$) in a waveguide filled by a low-density plasma ($n < n_{LH}$, $\varepsilon_\perp > 1$), with z normal to sheaths and parallel to **B**.

In the electrostatic (ES) approximation, the eigenmode equation is $\nabla \cdot \mathbf{E} = -\nabla^2 \Phi = 0$ and the solution is obtained using the ansatz

$$\Phi(z) = \Phi_0 \cos k_{||} z\; e^{i k_x x}$$

$$E_x(L) = -i k_x \Phi_0 \cos(k_{||} L)\; e^{i k_x x} = -i k_x\; \Phi(L)\;\; ,$$

$$E_z(L) = k_{||} \Phi_0 \sin(k_{||} L)\; e^{i k_x x} = k_{||} \tan(k_{||} L)\; \Phi(L)\;\; ,$$

Our sheath BC imposes following constraint on $k_{||}L$:

$$(k_{||} L) \tan(k_{||} L) = \frac{L}{\Delta} \frac{\varepsilon_{zz}^{sh}}{\left|\varepsilon_{||}^{p}\right|} \equiv \frac{(1 + i\nu)}{\Lambda}$$

where $\Lambda = \left|\varepsilon_{||}^{p}\right| (\Delta / L) = (\omega_{pe}^2 / \omega^2)(\Delta / L)$. In the ES limit, the slow wave dispersion relation $n_\perp{}^2 \varepsilon_\perp + \left(n_{||}{}^2 - \varepsilon_\perp\right)\varepsilon_{||} = 0$ implies

$$k_\perp{}^2 = \frac{k_{||}{}^2\, \omega_{pe}^2}{\varepsilon_\perp\, \omega^2}$$

so that dissipation ($\Rightarrow$ complex $k_{||}$) will lead to evanescence ($\Rightarrow$ complex $k_\perp = k_x$) as the mode propagates across the field. Finally, the sheath voltage is given by

$$V_{sh} = \Phi(L) = \Phi_0 \cos(k_{||} L)\, e^{i k_x x}$$

These eqs. together with the relations for Δ, ν and P_{sh} form a complete system that can be solved simultaneously (not yet done).

SOLT 2D Code

SOLT (Scrape-Off Layer Turbulence) is a 2D code that solves for plasma evolution on the slow time scale of **turbulent transport** ($\omega \ll \Omega_i$) including the interaction with **nonlinear rf effects**. In 2D, this code unifies:

- physics of SOL turbulence (blob production)
- blob transport
- rf-sheath-convection
- ponderomotive density expulsion (not yet added)

for the purposes of studying rf-turbulence interactions and for computing the self-consistent SOL density profile.

The code was developed originally for blob studies and solves the field-line-averaged vorticity, density and temperature equations in the poloidal plane (x,y). As part of the STTR project, it was modified to include models of the rectified rf sheath potential Φ_0 and its associated time-averaged parallel current:

$$\frac{\langle J_{||} \rangle}{nec_s} = 1 - \upsilon\, e^{-\Phi / T} I_0(\xi)$$

$\Rightarrow$ rectified potential $\exp[\Phi_0/T_e] = \upsilon\, I_0(\xi)$,

where $\upsilon = (m_i/2\pi m_e)^{1/2}$, $\xi = ZeV_{sh}/T_e$.

The antenna and Faraday screen structure give a spatial dependence $\Phi_0(x,y)$ that drives **E**×**B** convective cells. This is modeled by

$$V_{sh}(x, y) = V_{sh}(x, y = 0)\ [f + (1 - f)\cos(k\, y)]$$

where f is a flux cancellation factor (0 < f < 1) depending on the antenna phasing, and k is determined by the poloidal periodicity of the antenna, e.g. the modulation of the rf flux by the Faraday screen.

Initial Results of SOLT code + rf

● turbulent density profile as a function of antenna voltage:

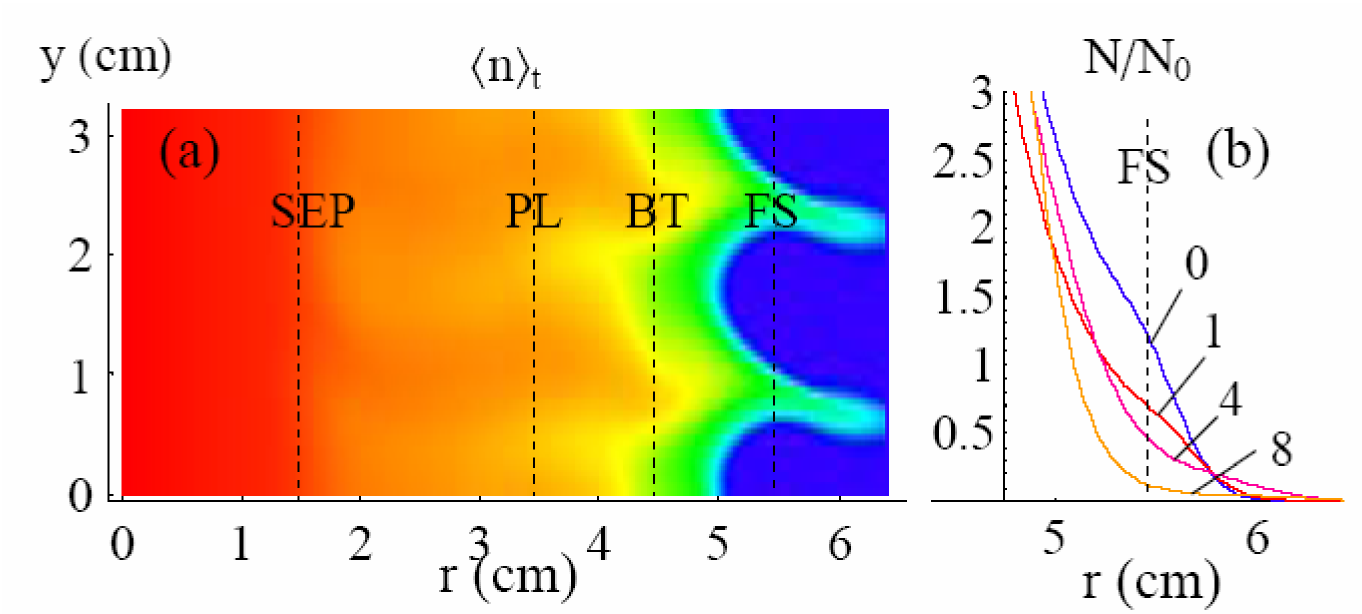


Fig. 1 (a) Time-averaged turbulent density fluctuations from a SOLT simulation as a function of radial (r) and poloidal (y) variables, and (b) poloidally- and temporally-averaged density profiles, $N(r) \equiv \langle n(r,y,t)\rangle_{y,t}$, from 4 simulations: V(FS)/Te = 0 (antenna off), 1, 4 and 8 (shown in (a)), where V(FS) is the antenna voltage at the Faraday screen. The density decreases at the FS with increasing antenna voltage, though the flux increases there. SEP, PL and BT denote the radial locations of the separatrix, poloidal limiter and bumper tile tips, respectively.

• interaction of turbulent blobs with rf convective cells:

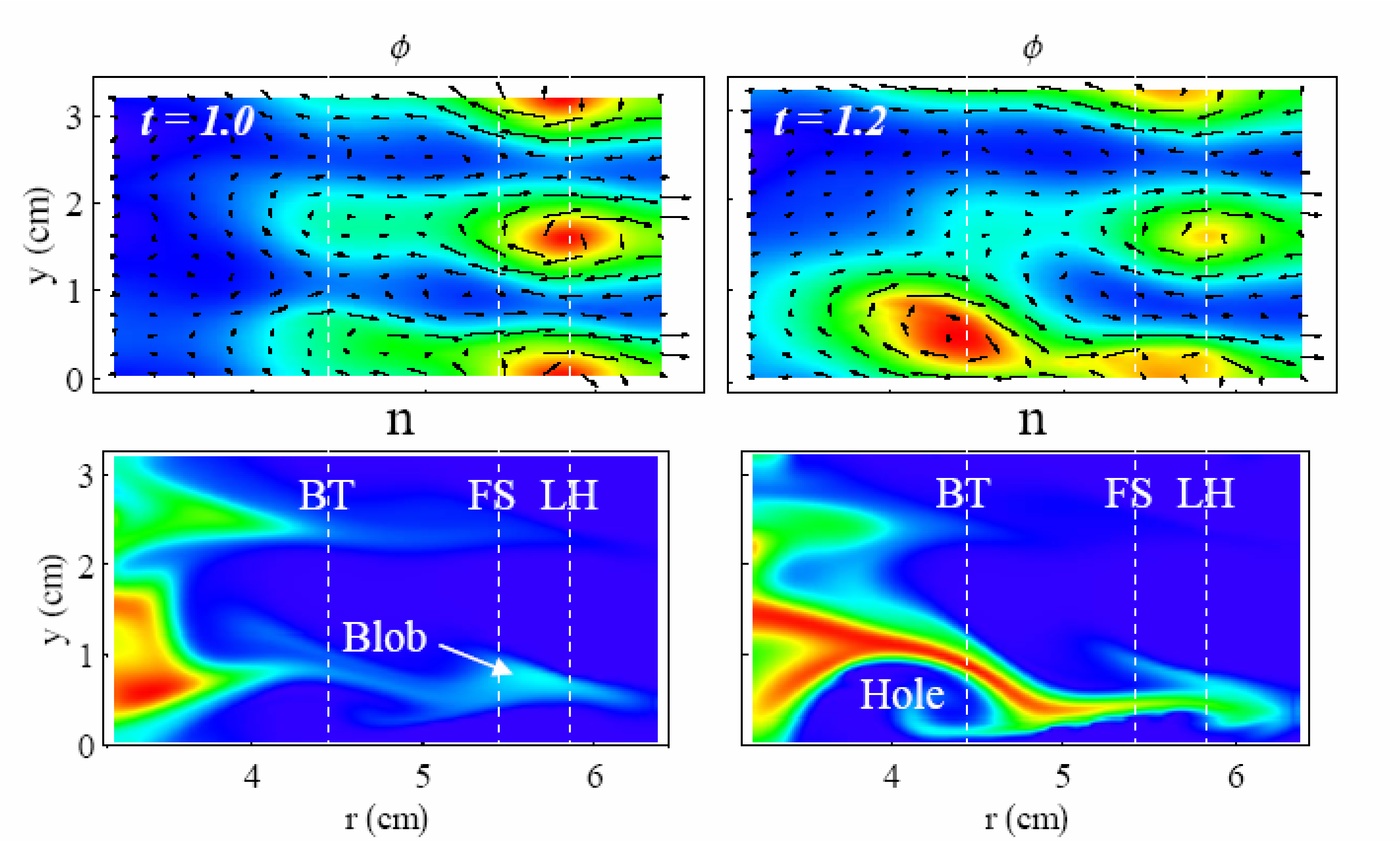


Fig. 2 Snap-shots of electrostatic potential ϕ and plasma density n as functions of radial and poloidal variable, r and y respectively, from the SOLT simulation based on the MORRFIC sheath potential $V_1(r)$ in Fig. 3 below. LH denotes the lower hybrid resonance, based on $N_0(r)$. Arrows denoting the convecting **ExB** velocity overlay the ϕ plot, where $\mathbf{E} = -\nabla\phi$. Notice the pronounced convection of the density blob into canals between the isolated vorticity cells associated with the LH resonance. Also note the large vorticity cell shed by the antenna, correlated with a density depression or *hole*.

Note:

- turbulence dominates for r < 4 cm**;**
  rf convection dominates for r > 4 cm
- blob transports outwards, squeezed between rf convective cells; density hole interacts with rf convective cell and moves inwards.

Code integration and future plans

We have done a preliminary iteration of SOLT and MORRFIC:

1. SOLT outputs the density profile <n(x)> for an assumed initial rf configuration. <Q> $\Rightarrow$ average Q over y and t.
2. The "radial-toroidal" version of MORRFIC inputs <n(x)> from SOLT, computes the rf fields, and outputs the two nonlinear functions $V_{sh}(x,y)$ = rf sheath voltage and $\Psi(x,y)$ = ponderomotive potential. (V_{sh} is computed from the *vacuum* sheath BC.)
3. SOLT inputs $V_{sh}(x,y=0)$ used in FS model, recomputes the density profile including rf convection, and outputs an updated <n(x)> to MORRFIC.
4. repeat steps 2 and 3 manually

In the next phase, the codes will be generalized to include:

a) SOLT:

- add two-region model (rf region with convective and ponderomotive force effects, non-rf particle source region upstream)
- add ponderomotive force ($\|$ and $\perp$) effects
- import 2D profiles, $V_{sh}(x,y)$ and $\Psi(x,y)$, from MORRFIC

b) MORRFIC:

- use *analytic* sheath BC and field-line mapping algorithm $\Rightarrow V_{sh}(x,y)$ and $P_{sh}(x,y)$; add visualization diagnostics for "hot spots"
- automate code iteration
- import n(x,y) from SOLT to "radial-poloidal" version of MORRFIC for studies of wave scattering off turbulent density / blobs.

Preliminary results: code iteration

- results of one "open loop" iteration of MORRFIC and SOLT:

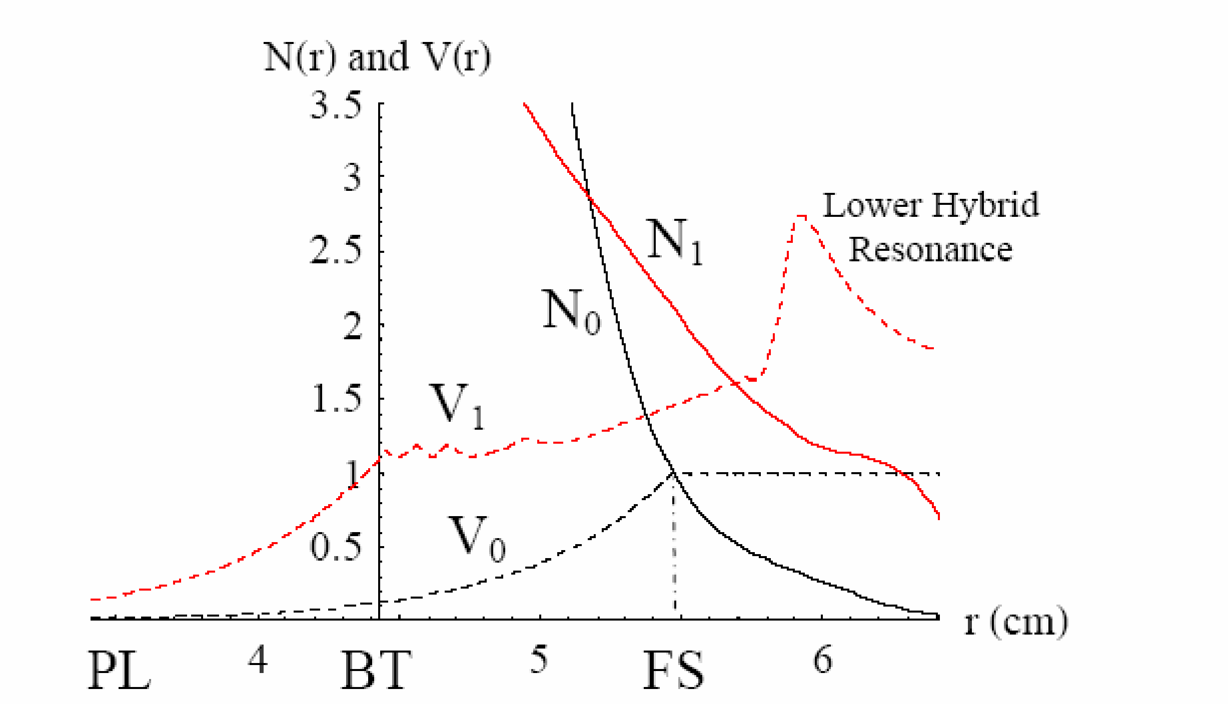


Fig. 3 Antenna voltage V(r) and plasma density profile N(r) plotted as functions of radius near the antenna (PL = poloidal limiter, BT = bumper tiles (antenna limiter), FS = Faraday Screen). V_0 and N_0 are initial profiles (black), and V_1 and N_1 are the profiles after one iteration of the MORRFIC-SOLT loop (red). (N and V are normalized to N_0(FS) and V_0(FS).) The separatrix is at r = 1.4 cm. Notice the density enhancement behind the FS, apparently associated with the lower hybrid resonance that MORRFIC discovered given N_0.

Note:

- density enhancement and LH resonance behind FS
- density depression in front of antenna (due to rf convection)

Ponderomotive Force Model

We have derived (but not yet implemented) a set of generalized model equations for the 2D SOLT code including two regions (rf, particle source) and effects of the ponderomotive force **F**. The basic equations are

$$\frac{d}{dt}\left(\frac{nmc^2}{B^2}\nabla_\perp^2\Phi\right)=\nabla_{||}J_{||}+\frac{2c}{B}\mathbf{b}\times\boldsymbol{\kappa}\cdot\nabla p+\frac{c}{B}\mathbf{b}\cdot\nabla\times\mathbf{F}$$

$$\eta_{||}J_{||}+\nabla_{||}\Phi=\frac{1.71}{e}\nabla_{||}T_e+\frac{T_e}{en}\nabla_{||}n+\mathbf{F}_{e||}$$

$$nm\frac{dv_{||}}{dt}=-\nabla_{||}p+F_{||}$$

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{v})=0$$

where the PF is $\mathbf{F}=-n\nabla\Psi+\mathbf{B}\times\nabla\times\mathbf{M}$ in the cold-fluid limit and $\Psi=\Psi_e+\Psi_i$ is the ponderomotive potential, e.g. $\Psi_e=m_e\left(e\left|E_{||}\right|/2m_e\omega\right)^2$.

Notes:

(1) in the "short antenna" limit ($\tau_{||ant}=L_{||ant}/c_s$ short compared to other time scales) and isothermal (T = const) approximation, the parallel PF gives the usual result for **ponderomotive density expulsion** in front of the antenna:

$$n=n_0\exp\left(-\Psi/\left(T_e+T_i\right)\right)$$

(2) the perpendicular PF in the vorticity equation produces **charge separation** (similar to the curvature force) and affects the blob motion (shields antenna).

PF Model (cont.)

Let $L_{||}$ be the length of field lines passing in front of an antenna of length $L_{||ant}$. For general $L_{||}/L_{||ant}$, treating PF effects requires a two-region model (rf and non-rf regions), but can analytically reduce the equations if $L_{||ant} << L_{||}$.

- Assume an intermediate time scale τ such that

$$\tau_{||ant} << \tau << \tau_{||}$$

(PF effect faster than blob motion, which is faster than parallel flow)

- Use quasistatic, isothermal parallel force balance (i.e. Boltzmann) for the parallel variation of n and Φ, and line-average the vorticity equation for the perpendicular dynamics. The desired **unification of blob and ponderomotive physics** in the vorticity equation is given by

$$\frac{d}{dt}\left(\frac{mc^2}{B^2} n \nabla_\perp^2 \Phi\right) = \frac{J_{||bc}}{L_{||}} + \frac{2c}{B}\mathbf{b}\times\boldsymbol{\kappa}\cdot\nabla p + \frac{L_{||ant}}{L_{||}}\frac{c}{B}\mathbf{b}\cdot\nabla\Psi\times\nabla\left(ne^{-\Psi/(T_e+T_i)}\right)$$

where $\mathbf{v} \approx \mathbf{v}_E$ (i.e. the PF drift is small compared to **E**×**B** drift) when

$$\frac{\Psi}{T}e^{-\Psi/T} < \frac{L_{||}}{L_{||ant}} \quad .$$

The PF term is important (compared to curvature) in the vorticity eq. when

$$\boxed{\frac{\Psi}{T}e^{-\Psi/T} > \frac{qa}{L_{||ant}}}$$

In this limit the PF can influence the blob motion.

Conclusions

● A collaboration between ORNL and Lodestar has begun developing a suite of codes for quantitative modeling of ICRF-edge plasma interactions and for studying the interplay between rf and SOL turbulence.

● The physics ingredients are mostly well-known but interact in complicated ways ⇒ need for an integrated numerical approach

● There is good motivation and opportunity for progress in this area:

- ➢ motivation: prospect of a burning plasma experiment (motivation)
- ➢ opportunity: funding for grand challenge computing (e.g. rf SciDAC and Fusion Simulation Projects)

● So far we have developed the following elements of the model:

- o 2D MORRFIC antenna coupling code (ORNL)
- o 2D SOLT turbulence and transport code (Lodestar)
- o rf sheath and ponderomotive models for the transport code
- o rf sheath BC for the rf antenna code

and we have demonstrated "open loop" coupling of the codes.

● Preliminary results have been obtained:

- ➢ interesting and complex interaction between turbulence and rf has been observed
- ➢ density profile is sensitive to the iteration of the coupled codes

● The next steps are

- o upgrade the physics models in each code
- o automate the iteration process
- o add far field sheaths and full-wave codes (long term: RF SciDAC)