thoughts on:

RMP-Induced Magnetic Shear and Implications for Stability, Blob Transport and Radial Electric Fields

J. R. Myra, D. A. D'Ippolito and D. A. Russell

Lodestar Research Corp., Boulder, CO

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Outline

- Introduction
- Instability and blob physics in axisymmetric tokamaks
- RMP-induced shear
- $E_\perp$ penetration in stochastic fields: the micro-scale problem
- E_r damping in stochastic fields: the macro-scale problem
- Conclusions

Introduction

experiments

- on DIII-D resonant magnetic perturbations (RMP) can
  - stabilize ELMs
  - increase radial particle transport (in low collisionality regimes)
  - modify E_r in the edge plasma
- other experiments show both similar and different effects
  - profile modifications of $T_e(r)$ vs. $n_e(r)$

theory

- RMP (stochastic) fields “mix” SOL and edge
- important edge physics for instabilities, turbulence, blob transport
  - sheath and presheath potentials, Reynolds stress and E_r
  - magnetic shear and parallel “disconnection”

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Edge/SOL "blob" ordering

Krasheninnikov, D'Ippolito, Myra, 2008, blob review

- vorticity

$$\nabla \cdot \frac{d}{dt}\left(\frac{nMc^2}{B^2} \nabla_\perp \Phi \right) = \nabla_{||} J_{||} + \ldots \qquad J_{||sh} = nec_s \left(1 - e^{e(\Phi - \Phi_B)/T} \right)$$

- density

$$\frac{\partial n}{\partial t} + \mathbf{v}_E \cdot \nabla n = D \nabla^2 n - \frac{n}{\tau_{||n}} + \ldots$$

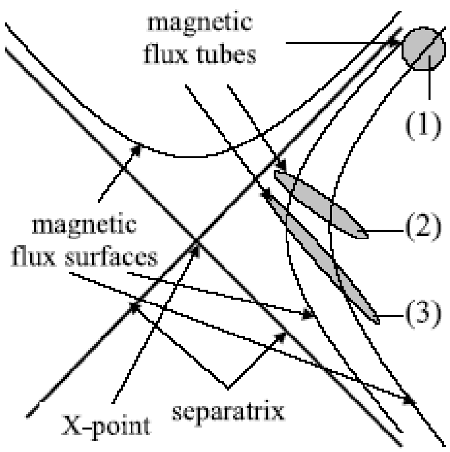
- electron temperature

$$\frac{\partial T}{\partial t} + \mathbf{v}_E \cdot \nabla T = \chi_\perp \nabla^2 T - \frac{T}{\tau_{||T}} \ldots$$

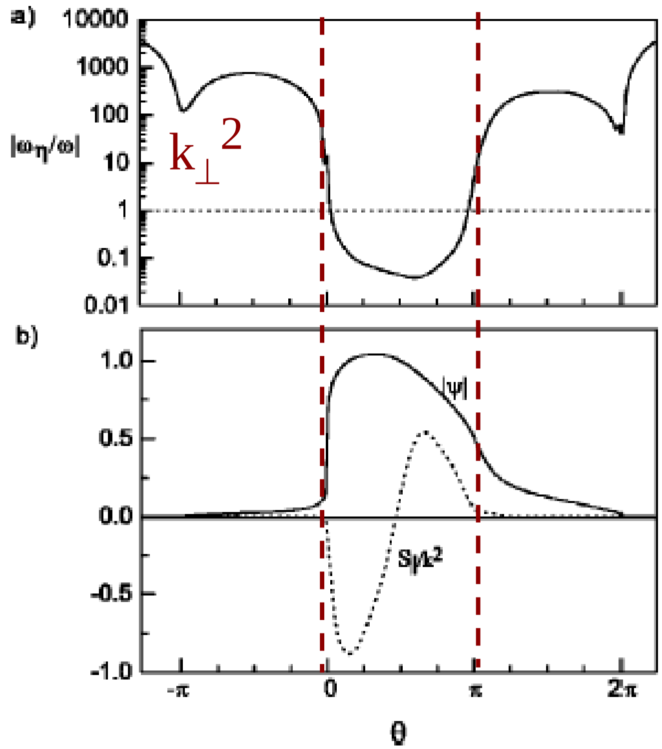
- *sheath & parallel physics important for potential and T_e*
- *perpendicular convective transport important for density*

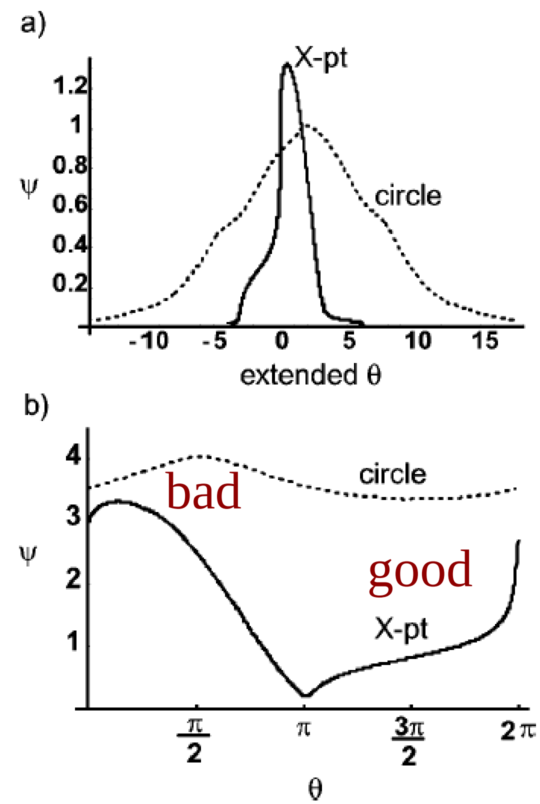
Disconnection and resistive X-point modes

- X-pt induced shear enhances local $k_\perp$
- allows parallel resistive disconnection of modes
  - modes can localize to maximize "bad" curvature
  - increases γ_{lin} and turbulence
- similar phenomenon for blob-filaments [Ryutov; Krasheninnikov; Russell]



Krasheninnikov 2004
Farina, Ryutov 1993





Myra, D'Ippolito, Xu & Cohen 2000

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Model: field line equations and map

- circular flux surfaces (r, θ) with q = q(r), B = const
- ψ(r, θ) = RMP perturbation

$$\frac{dr}{d\zeta} = \frac{R}{rB} e^{in\zeta} \frac{\partial\psi}{\partial\theta} \qquad \frac{d\theta}{d\zeta} = \frac{1}{q} - \frac{R}{rB} e^{in\zeta} \frac{\partial\psi}{\partial r}$$

- integrate unperturbed orbits over one toroidal transit

$$r_1 = r_0 + \frac{1}{r_0}\frac{\partial J_0}{\partial\theta_0} \qquad \theta_1 = \theta_0 + \frac{2\pi}{q_0} + \frac{1}{r_0}\frac{\partial}{\partial r_0}\left(\frac{2\pi}{q_0}\right)\frac{\partial J_0}{\partial\theta_0} - \frac{1}{r_0}\frac{\partial J_0}{\partial r_0}$$

$$J_0 = \frac{R}{B} \mathrm{Re} \int_0^{2\pi} d\zeta\, e^{in\zeta}\, \psi(r_0, \theta_0 + \zeta / q_0)$$

- small parameter

$$\frac{1}{r^2}\frac{\partial J}{\partial\theta} \sim \frac{mR}{r}\frac{B_r}{B}$$

- map preserves area through first order in J

Resonant perturbations and the standard map

$$\psi = \sum_m \psi_m e^{-im\theta}$$

$$J_0 = \sum_m \mathrm{Re}\frac{R\psi_m}{B} e^{-im\theta_0} \frac{e^{i\xi}-1}{i\xi} \to \text{const in } r$$

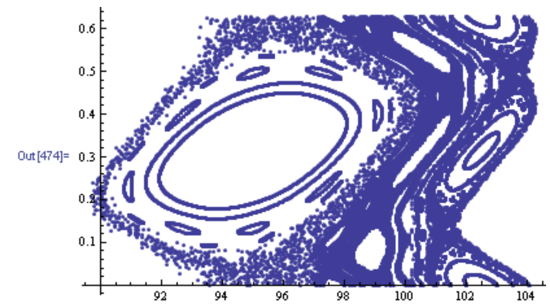
$$\xi = n - m/q_0$$

$$J_0 \to \frac{2\pi R\psi_m}{B}\cos m\theta_0 \equiv H\cos m\theta_0$$

$$r_1 = r_0 + \frac{K\sin m\theta_0}{m\iota'}$$

$$\theta_1 = \theta_0 + \iota_0 + \frac{K\sin m\theta_0}{m}$$

$$\iota_0 = \frac{2\pi}{q_0}$$



$$K = -\frac{m^2 H\iota'}{r_0} = \text{Chirikov parameter}$$

- for standard map in canonical form

$$\iota = \iota' r \qquad p = m\iota' r$$

$$\iota' = \text{const} \qquad \varphi = m\theta$$

$$K = \text{const}$$

$$p_1 = p_0 + K_0 \sin\varphi_0$$

$$\varphi_1 = \varphi_0 + p_1$$

RMP-induced magnetic shear

- evaluate metric (Jacobian) of the map

$$M = \begin{pmatrix} \partial r_1 / \partial r_0 & (1/r_0)\partial r_1 / \partial \theta_0 \\ r_1 \partial \theta_1 / \partial r_0 & (r_1/r_0)\partial \theta_1 / \partial \theta_0 \end{pmatrix} = \begin{pmatrix} 1 & \frac{K}{\iota' r}\cos m\theta \\ \iota' r & 1 + K\cos m\theta \end{pmatrix}$$

- mapping of $k_\perp$ given by $\mathbf{k}_1 = M^{tr}\mathbf{k}_0$

$$\begin{pmatrix} 1 & \iota' r \\ \frac{K\cos m\theta}{\iota' r} & 1 + K\cos m\theta \end{pmatrix} \begin{pmatrix} k_r \\ k_\theta \end{pmatrix} = \begin{pmatrix} k_r + \iota' r\, k_\theta \\ \frac{K\cos m\theta}{\iota' r} k_r + (1 + K\cos m\theta) k_\theta \end{pmatrix}$$

nb: $K = 0 \Rightarrow$ usual $\perp$ eikonal ballooning formalism

- RMP-induced shear dominates background magnetic shear *for a single transit* when $\iota' r < K$

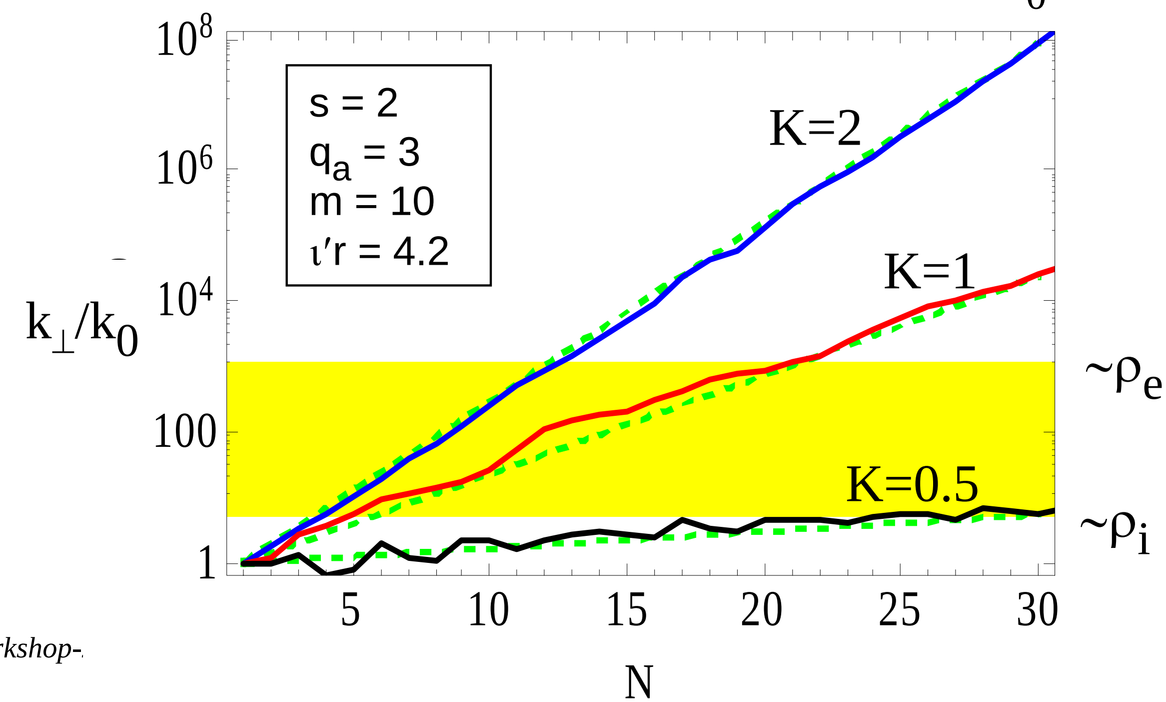
RMP-induced magnetic shear

- two applications of the map (2 transits): background shear increases linearly, RMP shear exponentially

$$M^{tr} \cdot M^{tr} = \begin{pmatrix} 1 + K\cos m\theta & \iota' r(2 + K\cos m\theta) \\ \dfrac{K\cos m\theta(2 + K\cos m\theta)}{\iota' r} & K\cos m\theta + (1 + K\cos m\theta)^2 \end{pmatrix}$$

- N >> 1 applications, $\langle\cos^2\theta\rangle \to$ ½, K >> 1

$$\frac{k_\perp^2}{k_0^2} \approx \frac{K^{2(N-1)}}{2^{(N-1)}}\left[1 + \frac{1}{2}\left(\frac{K}{\iota' r}\right)^2\right]$$



Implications for ballooning stability

- for modes which wrap around torus many times (extended ballooning angle >> 2π) RMP-induced-shear effects will dominate and limit mode extension
  - global shear (resonant RMP) usually stabilizing for ideal modes
- local shear (non-resonant RMP) can be destabilizing for resistive modes
  - disconnects blobs from good curvature
  - enhanced transport (density pump-out)
  - related work:
    - Waltz & Boozer, (1993)
    - Hegna and Hudson (2001)
    - Beyer et al (1998, 2002)
    - Xu (2007)
    - Reiser (2005)

analogous to X-pt resistive disconnection

- needs quantitative work

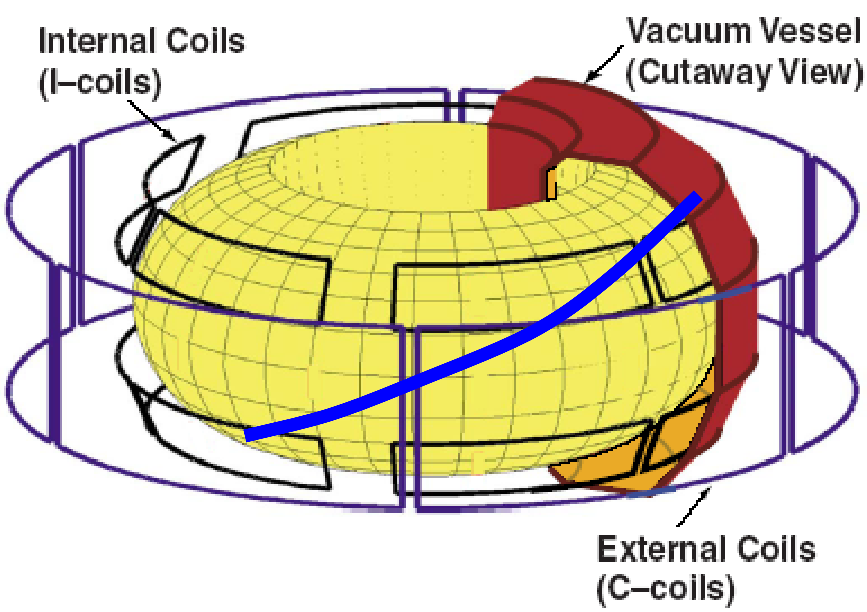


Fig. from Fenstermacher 2008

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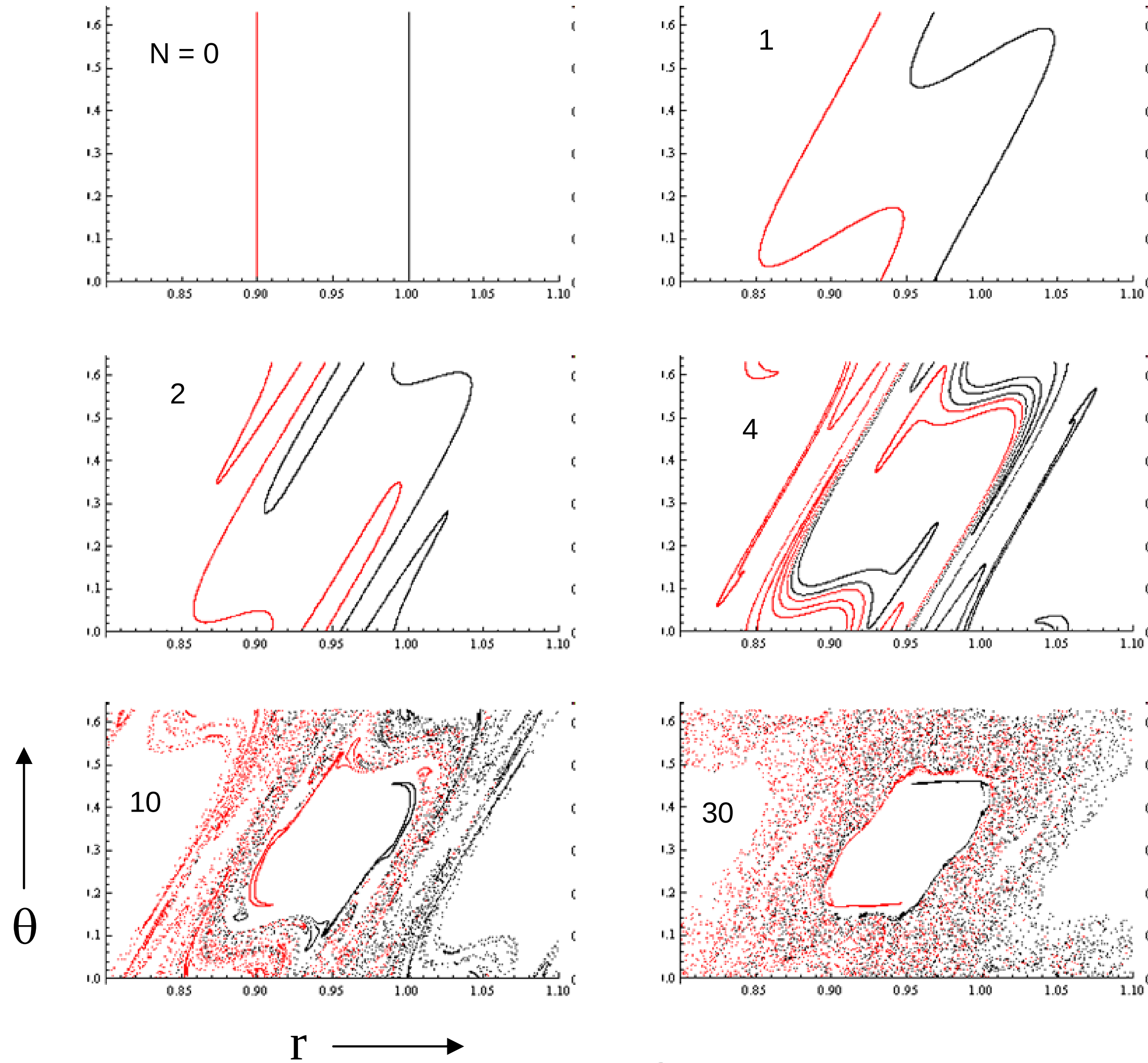
Penetration of potentials in stochastic fields

- evolution of 2 equi-potential surfaces after N transits
- fine scale structure develops
- surfaces become close ⇒ easily shorted by $\sigma_\perp$
- inter-diffusion of surfaces on long space scales
- kick per transit $\delta r \sim 2\pi R\, \delta b_r$ separates scales
  - micro-scale $\Delta x < \delta r$
  - macro-scale $\Delta x > \delta r$

$$\Delta x = (D_m L_{\|})^{1/2}$$



K = 2

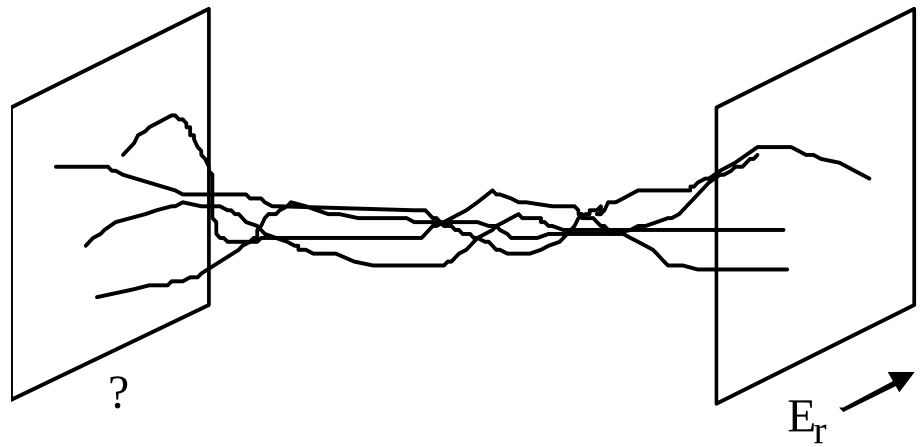
Micro-scale problem

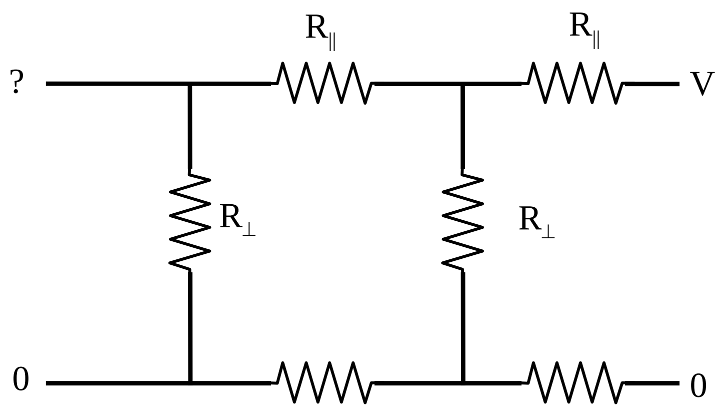
What is the parallel penetration length of an applied $E_\perp$?

- (pre)-sheath potential connecting into core

- core E_r connecting into SOL

- 2 planes are linked by a stochastic map
- $E_\perp$ is applied on the right plane
- calculate it on the left.





circuit model same as for X-point effects:

–RX mode

–blobs

see also: Kaganovich, PoP, 1998

Myra & D'Ippolito, PoP (2005)

Vorticity equation for penetration length

$$\sigma_\perp \nabla_\perp^2 \Phi + \sigma_\| \nabla_\|^2 \Phi = 0$$

$$k_\perp^2 \approx k_0^2 (C_0 + K^2/2)^N \qquad N = \frac{s}{2\pi R} \qquad s_0 = \frac{2\pi R}{\ln(C_0 + K^2/2)}$$

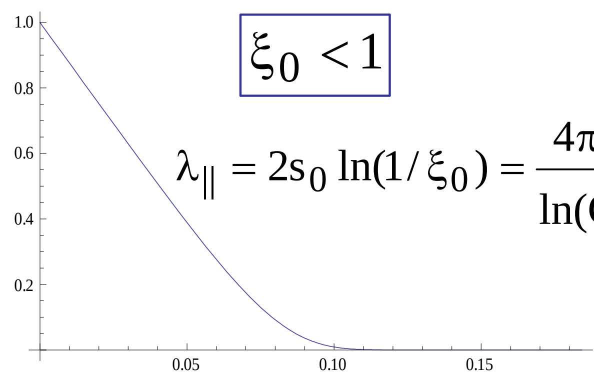
$$\xi = k_0 s \left(\frac{\sigma_\perp}{\sigma_\|} \right)^{1/2} \qquad \xi_0 = k_0 s_0 \left(\frac{\sigma_\perp}{\sigma_\|} \right)^{1/2}$$

$$\frac{d^2\Phi}{d\xi^2} = e^{\xi/\xi_0}\Phi \quad \Rightarrow \quad \Phi = \Phi_0 \frac{K_0(2\xi_0 e^{\xi/2\xi_0})}{K_0(2\xi_0)}$$

$\xi_0 < 1$

$$\lambda_\| = 2 s_0 \ln(1/\xi_0) = \frac{4\pi R \ln(1/\xi_0)}{\ln(C_0 + K^2/2)}$$

Φ

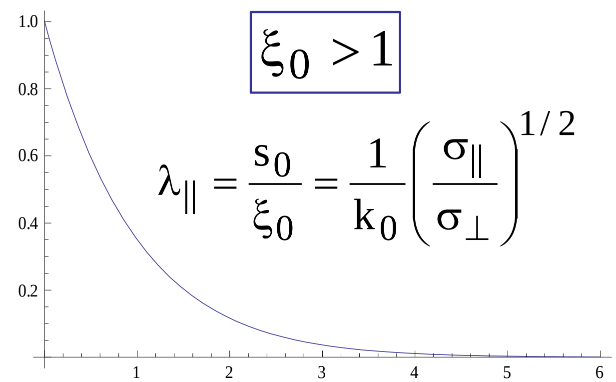


ξ

$\xi_0 > 1$

$$\lambda_\| = \frac{s_0}{\xi_0} = \frac{1}{k_0} \left(\frac{\sigma_\|}{\sigma_\perp} \right)^{1/2}$$

Φ



ξ

distance along B →

Perpendicular conductivity

- ion polarization drift
  - note that area preservation of map ⇒ $\mathbf{v}\cdot\nabla$ invariant $\mathbf{v} = \frac{c}{B}\mathbf{b}\times\nabla\Phi$

$$\sigma_\perp \sim \frac{c^2}{4\pi v_a^2}\mathbf{v}\cdot\nabla \sim \frac{c^3 k_0^2 \Phi}{4\pi v_a^2 B} \sim \frac{e\Phi}{T} k_0^2 \rho_s^2 \frac{\omega_{pi}^2}{4\pi\Omega_i}$$

  - applies on scales $L_\perp > \rho_i$
- electron collisional conductivity [Ryutov & Cohen 2004 (X-pts & blobs)]
  - on scales $\rho_e < L_\perp < \rho_i$ ions can't respond to $E_\perp$ but electrons can

$$\sigma_\perp = \frac{\omega_{pe}^2 \nu_{ei}}{4\pi\Omega_e^2}$$

Parallel decay length

$$\xi_0 = k_0 s_0 \left(\frac{\sigma_\perp}{\sigma_\parallel}\right)^{1/2} < 1 \qquad \Rightarrow \qquad \lambda_\parallel = 2 s_0 \ln(1/\xi_0) \qquad \text{where} \qquad s_0 = \frac{2\pi R}{\ln(1+K^2/2)}$$

$$\left(\frac{\sigma_\perp}{\sigma_\parallel}\right)^{1/2} = \begin{cases} k_0 \rho_s \left(\dfrac{e\Phi}{T} \dfrac{\nu_{ei}}{\Omega_e}\right)^{1/2} & \text{ion regime} \\ \dfrac{\nu_{ei}}{\Omega_e} & \text{electron regime} \end{cases}$$

- e.g. for electron regime

$$\lambda_\parallel = \frac{4\pi R}{\ln(1+K^2/2)} \ln\left(\frac{\rho_i \Omega_e}{s_0 \nu_{ei}}\right)$$

  - $\ln(\rho_i\, \Omega_e / s_0 \nu_{ei}) \sim 4.6$ at 50 eV $\rightarrow$ 10 at 1keV

- typically, micro-scale $E_\perp$ is shorted out in a few toroidal transits
  - e.g. (pre)-sheath potential, core E_r

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Averaged vorticity equation describes macro-scale potential

$\nabla_{||} \rightarrow \nabla_{||0} + \delta\mathbf{b}\cdot\nabla_{\perp}$ QL average for $<|\delta b_r|^2>$

- integrate vorticity to get momentum (zonal flow equation)
  - for simplicity n = const

$$\frac{\partial}{\partial t}\left\langle v_y \right\rangle = -\frac{\partial}{\partial x}\left\langle v_x v_y \right\rangle - \gamma_s \left\langle v_y \right\rangle + 2\Omega_i^2 \rho_s \int^x dx \frac{1}{\left\langle L_{||}\right\rangle}\left(\left\langle \frac{e\Phi}{T}\right\rangle - 3\right)$$

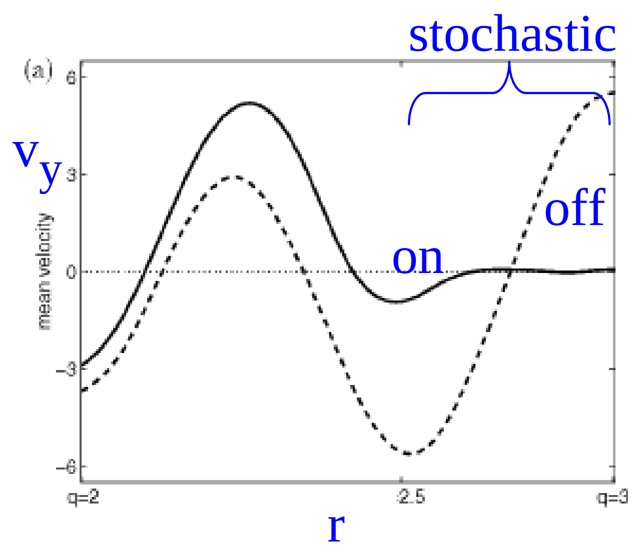
Reynolds stress — stochastic damping — sheath charge loss (heuristic)

- stochastic damping rate of zonal flows

$$\gamma_s = \frac{\Omega_e \Omega_i}{\nu_{ei}}\left\langle \left|\delta b_r\right|^2 \right\rangle \sim 10^6 \times 10^{-8}\Omega_i \sim 10^{-2}\Omega_i$$

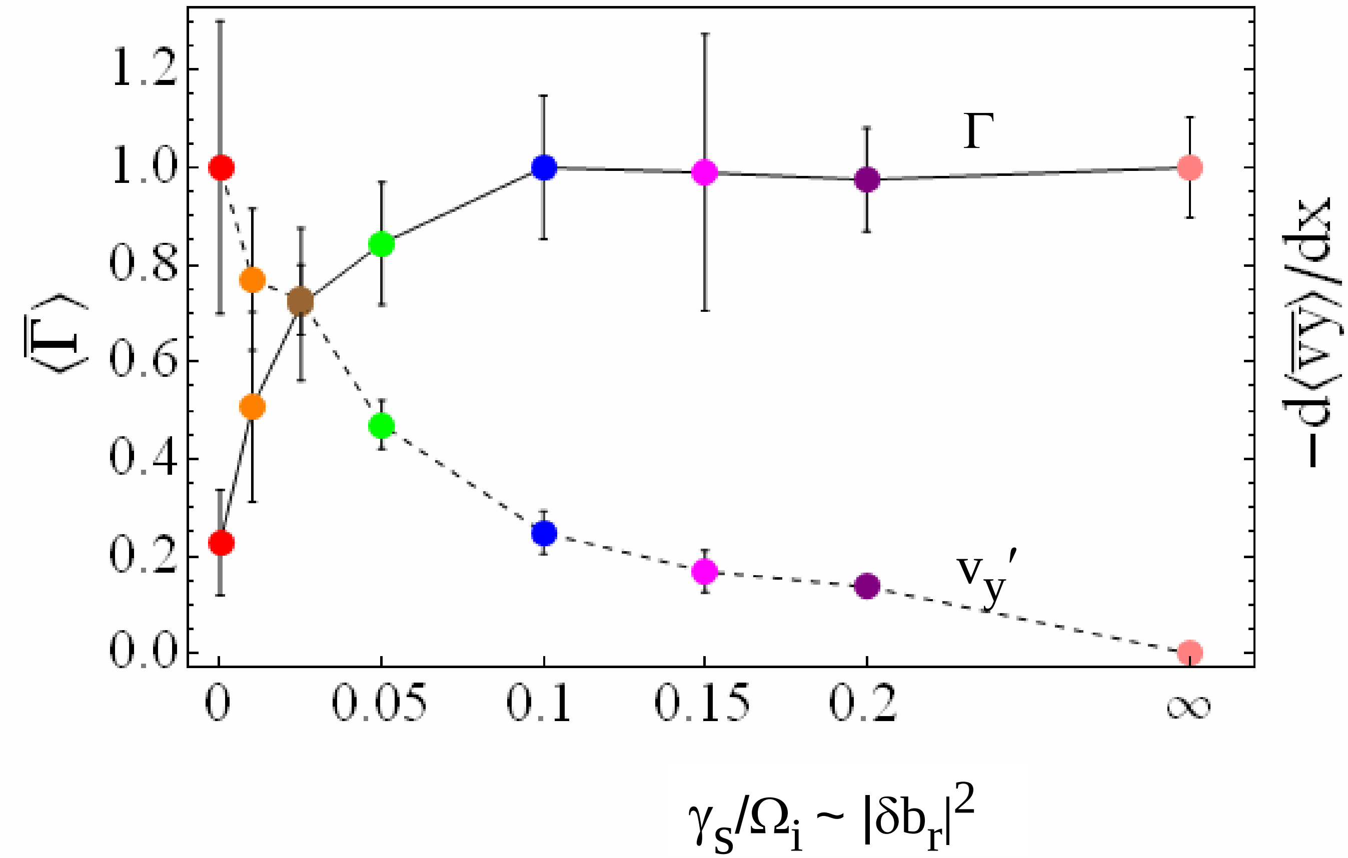
@ 100eV, 10^{13} cm^{-3},2T

  - significant, especially for collisionless plasma (but flux limited …)



Beyer, PPCF 2002

Strong zonal flow damping enhances turbulence and blob transport



SOLT turbulence code simulations [Russell et al. 2008; D'Ippolito IAEA 2008]

- for small γ_s, blob flux increases linearly with $\gamma_s \sim |\delta b_r|^2$

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Conclusions

- RMP-induced magnetic shear grows exponentially fast and tends to dominate multiple-transit phenomena
  - may provide one mechanism for parallel "disconnection" of unstable mode $\Rightarrow$ increased γ_{lin}, v_{blob}, and density pumpout (?); awaits quantitative evaluation
- disconnection physics also controls micro-scale $E_\perp$ in plasma
  - penetration of $E_\perp$ limited to a few toroidal transits
- stochastic damping of macro-scale E_r competes with both sheaths and Reynolds stress to limit zonal flows
  - theory predicts a strong reduction of E_r (zonal flow damping) in the presence of RMP
  - provides a second mechanism for enhanced perpendicular convective transport by turbulence and blobs (pumpout) in the presence of RMP
- preliminary results – much opportunity for interesting future work

Supplemental

Electron conductivity model

based on D.D. Ryutov and R.H. Cohen, Contrib. Plasma Phys. **44**, 168 (2004).

$$\mathbf{E} + \frac{1}{c}\mathbf{v}_e \times \mathbf{B} = \frac{\mathbf{R}_{ei}}{ne}$$

$$\mathbf{E} + \frac{1}{c}\mathbf{v}_i \times \mathbf{B} = \frac{\mathbf{R}_{ei}}{ne}$$

usually implies $v_e = v_i$ and $\mathbf{J}_\perp = 0$

(species friction forces equal and opposite)

- for small scales, $\rho_e < L_\perp < \rho_i$ ions, can't respond to $E_\perp$ but electrons can

$$\mathbf{v}_i = 0$$

$$\mathbf{J}_\perp = -ne\mathbf{v}_{\perp e} = \frac{nec}{B}\left(\mathbf{b} \times \mathbf{E} + \frac{\nu_{ei}}{\Omega_e}\mathbf{E}_\perp\right)$$

$$\sigma_\perp = \frac{nec}{B}\frac{\nu_{ei}}{\Omega_e} = \frac{\omega_{pe}^2 \nu_{ei}}{4\pi\Omega_e^2}$$

for $\nu_{ei} << \Omega_e$