

REDUCED-MODEL SIMULATIONS OF
TURBULENCE AND RF-DRIVEN CONVECTION
IN THE EDGE AND SCRAPE-OFF LAYER
PLASMA¹

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Introduction

- Strong turbulent fluctuations (**blobs**) in the SOL interact with sheath-generated E-fields of the **rf-antenna**; the antenna tends to impose a limiting, sheared flow pattern near the Faraday screen (FS).²
- We model this interaction in **2D** (outboard poloidal plane) using reduced **Braginskii** fluid equations supplemented with boundary conditions on $J_{//}$ appropriate for current closure in the **divertor sheath**, for the SOL, and in the **antenna sheath** at (and beyond) the FS.
- **Simulations of strong turbulence** are studied to assess the implications for **rf physics** (e.g. antenna/plasma interaction) and in an attempt to identify a control knob for **edge turbulence** (e.g. through rf-antenna-driven sheared **E**).

(2) D.A. D'Ippolito, et al., Phys. Fluids **B5**, 3603 (1993).

Reduced Braginskii Model Fluid Equations³ + RF Sheath Drive⁴

Vorticity

Plasma Velocity $\mathbf{v} = \mathbf{E} \times \mathbf{B} \, c/B^2$, $\mathbf{E} = -\nabla\phi$, so Vorticity $\sim \nabla^2\phi$

$$\begin{aligned} (\partial_t + [\phi, \circ] + \mathbf{v}) \nabla^2 \phi = & \alpha(x) \sqrt{T_e} \cdot \left[1 - \text{Exp}(\Lambda - \phi/T_e) \cdot I_0(V_{rf}/T_e) \right] + \\ & - \beta \partial_y (N \cdot T_e) / N + \mu \cdot \nabla^4 \phi \end{aligned} \quad (1)$$

Electron Density

$$(\partial_t + [\phi, \circ]) N = D \nabla^2 N - \alpha(x) N \sqrt{T_e} \cdot \text{Exp}(\Lambda - \phi/T_e) \cdot I_0(V_{rf}/T_e) + S_N(x) \quad (2)$$

Electron Temperature

$$(\partial_t + [\phi, \circ]) T_e = \chi \nabla^2 T_e - \alpha_T(x) T_e^{3/2} \cdot \text{Exp}(\Lambda - \phi/T_e) \cdot I_0(V_{rf}/T_e) + S_T(x) \quad (3)$$

$$[\phi, \circ] f \equiv \partial_x \phi \cdot \partial_y f - \partial_x f \cdot \partial_y \phi$$

Boundary Conditions:

$x = 0$ (edge):

$x = Lx$ (antenna):

Periodic in y

$$\phi = 0$$

$$\phi = \Lambda \cdot T_e + T_e \cdot \text{Ln} \left[I_0(V_{rf}/T_e) \right]$$

$$\nabla^2 \phi = 0$$

$$\nabla^2 \phi = \nabla^2(\text{above})$$

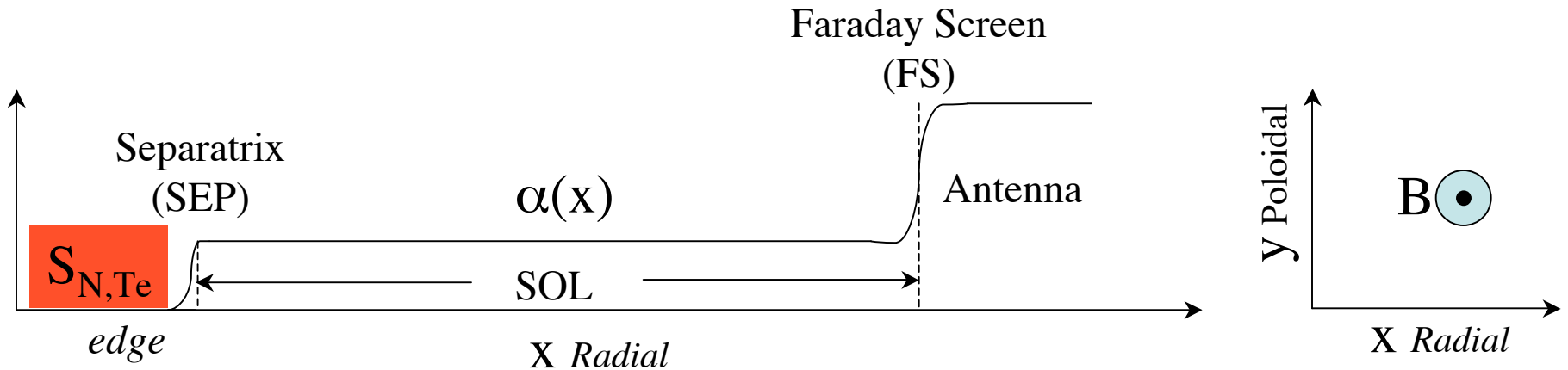
$$D \partial_x N = 0$$

$$N = N_F = \text{const.}$$

$$\chi \partial_x T_e = 0$$

$$T_e = T_F = \text{const.}$$

Orientation



In the outboard poloidal plane ($z = 0$), vorticity, electron density and temperature are evolved as above. The original fluid equations have been integrated along B-field lines, assuming invariance of all quantities, *except* J_{\parallel} which is assumed to be an odd function of z ; sheath boundary conditions are enforced on J_{\parallel} . A time average has been taken over the rf frequency that, for Boltzmann-distributed electrons, results in the Bessel function I_0 that appears in the sheath terms.⁴ The edge, SOL and antenna regions are distinguished by a radially dependent sheath coefficient, $\alpha(x)$, intended to simulate infinite, intermediate ($\sim 10\text{m}$) and short ($\sim 1\text{m}$) connection lengths in these regions respectively. N and T_e are driven by sources confined to the edge region.

- (3) See also: N. Bisai, et al., Phys. Plasmas **11**(8), 4018 (2004); O.E. Garcia, V. Naulin, A.H. Nielsen and J. Juul Rasmussen, Phys. Rev. Lett. **92**(16) 165003 (2004); V. Naulin, J. Juul Rasmussen and J. Nycander, Phys. Plasmas **10**(4), 1075 (2003).
- (4) D.A. D'Ippolito and J.R. Myra, Phys. Plasmas **3**(1), 420 (1996).

The Numerical Algorithm

Overall: **Fractional Time-Stepping**, e.g., Eq. 1 (Vorticity):

$$v_x = -\partial_y \phi; \quad (\partial_t + v_x \partial_x) \rho = 0; \quad -\nabla^2 \phi = \rho$$

$$v_y = \partial_x \phi; \quad (\partial_t + v_y \partial_y) \rho = 0; \quad -\nabla^2 \phi = \rho$$

$$\partial_t \rho = \mu \nabla^2 \rho; \quad (\partial_t + v) \rho = \alpha(\phi, T_e) - \beta \partial_y (NT_e) / N; \quad -\nabla^2 \phi = \rho$$

Convection: Lax-Wendroff⁵ (Up-Wind Differencing)

Diffusion (μ , D , χ): Crank-Nicholson⁶

Sources and Sinks (S_N , S_T , v , α , β , α_T): Explicit

Poisson Solver:

Fourier Synthesis with Cyclic Reduction *FACR(l)*⁷

- (5) W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, *Numerical Recipes in Fortran*, 2nd ed. Cambridge University Press (1992), p. 835.
- (6) Ibid., p. 840.
- (7) P.N. Schwarztrauber, *SIAM Review* **19**(3), 490 (1977); B.L. Buzbee, G.H. Golub and C.W. Nielson, *SIAM J. Numer. Anal.* **7**(4), 627 (1970).

Parameters of the Simulations

ν : Charge Dissipation, e.g. Neutral Friction, Alfvén Wave Emission

α : Sheath Coefficient, Discussion and figure above.

$\alpha_T = 8*\alpha$, here: Temperature relaxes faster than density.

ΔT_e : Bohm Potential

$\beta = 2\rho_s/R$: Curvature Drift Drives Charge Separation.

μ : Diffusion of Vorticity

S_N : Density Source, confined to edge region (figure above).

S_T : Temperature Source, confined to edge region.

$V_{rf}(x,y) = V_a * \text{Exp}[(x-x_0)/\delta] * [f+(1-f)*\cos(k_a*y)]$ (Ref.3)

Typically (unless noted otherwise):

$\nu = 0$, $\alpha_{\text{SOL}} = 5 \times 10^{-5}$, $\alpha_{\text{ANTENNA}} = 10 * \alpha_{\text{SOL}}$, $\Lambda = 3.9$

$\beta = 6.8 \times 10^{-4}$, $\mu = 0.1$

$S_N(x) = S_0 * \text{Exp}[(x/dx_0)^2] = S_T(x)$, $S_0 = 0.015$, $dx_0 = 8$

$D = 0.1$, $\chi = 0.1$, $N_F = 0.01$, $T_F = 0.01$

$V_a = 20 * T_F$, $x_0 = Lx$, $\delta = 4$, $f = 0.5$, $k_a = 2 * (2\pi/Ly)$

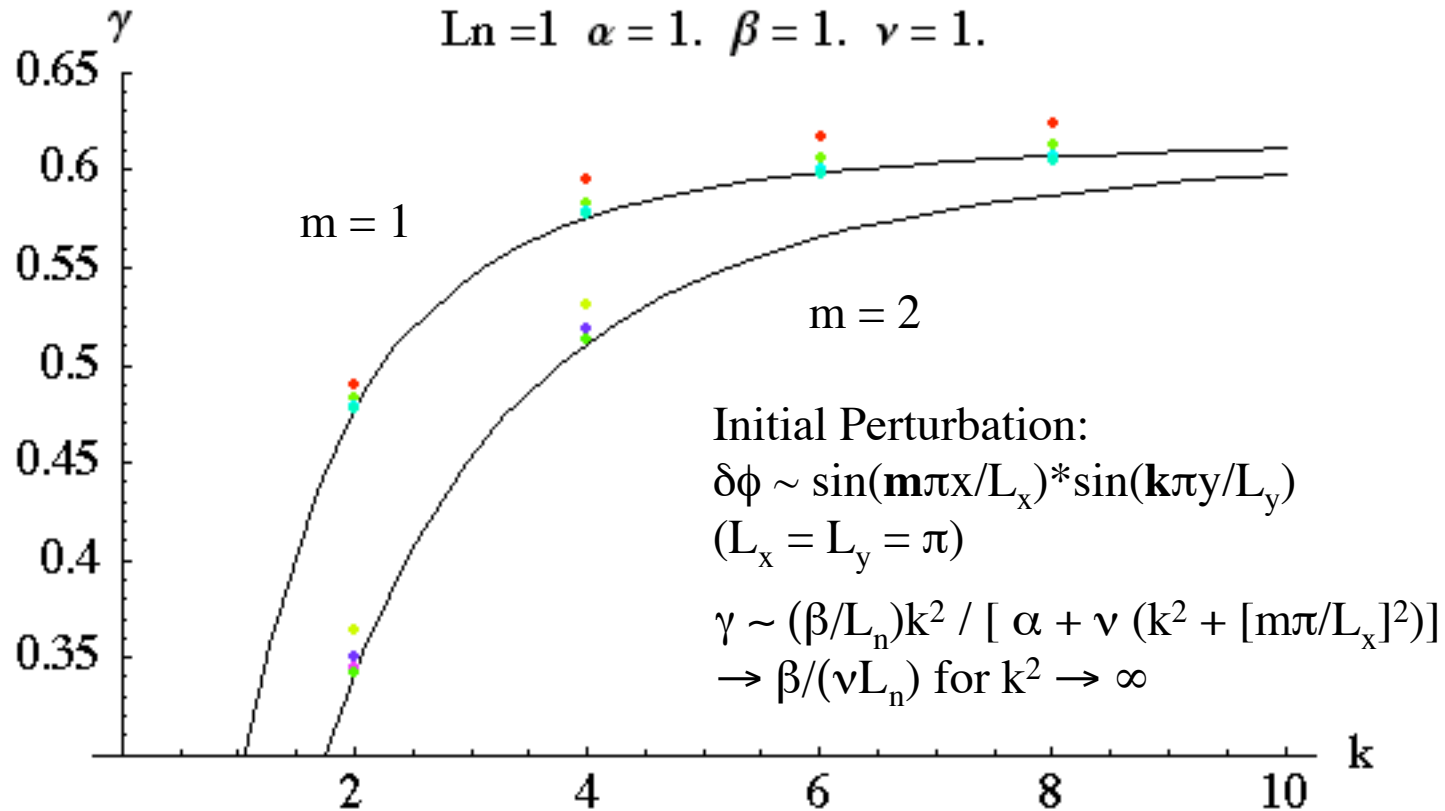
Physical Units:

$\rho_s = 1 \text{ mm}$, $\Omega_{ci} = 10^9 \text{ s}^{-1}$ (B = 3.3 T, Deuterium)

When dimensionless time changes by 1000, we've gone through 1 μs .

Benchmarking Exercise (1)

Linear Instability (Nedospasov / Rayleigh-Taylor)
of an Exponential Density Profile, $\text{Exp}(-x/L_n)$, $T_e = \text{constant}$.



Curves: Expected growth rate for $m = 1, 2$.

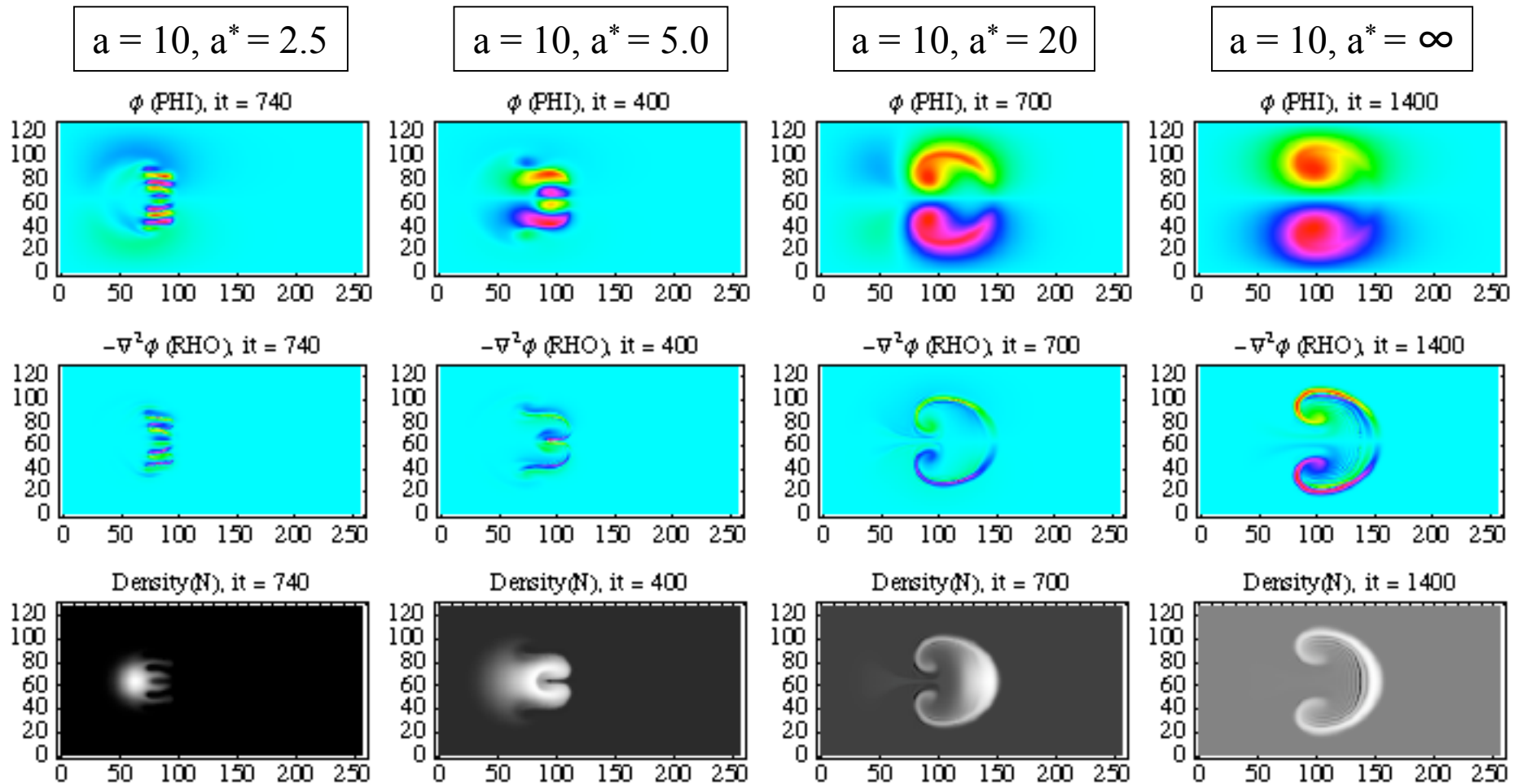
Dots: Observed growth rates, from numerical simulation of equations 1 and 2, converging to the expected result for successively reduced time steps: $dt = 0.1, 0.05, 0.025$ and 0.0125 .

Benchmarking Exercise (2)

Isolated blobs develop the expected instabilities⁽⁸⁾

$$N_0 \sim \text{Exp}[-r^2/(2a^2)] , a^* = (\beta/\alpha^2)^{1/5}$$

Rayleigh-Taylor \longrightarrow *Kelvin-Helmholtz*

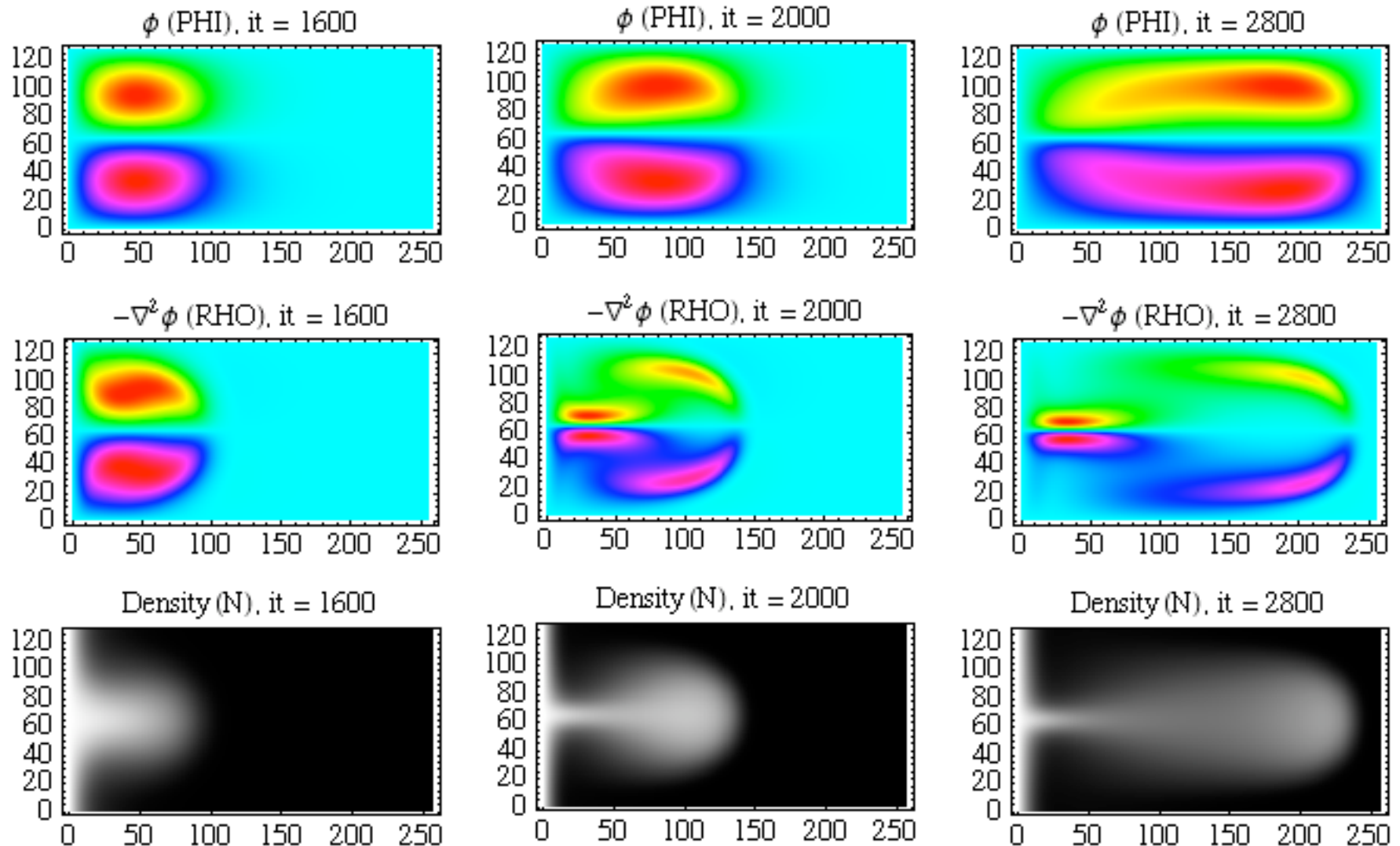


- (8) D.A. D'Ippolito and J.R. Myra, Phys. Plasmas **10**(10), 4029 (2003);
G.Q. Yu and S.I. Krasheninnikov, Phys. Plasmas **10**(11), 4413 (2003).

Here, $T_e = \text{constant}$.

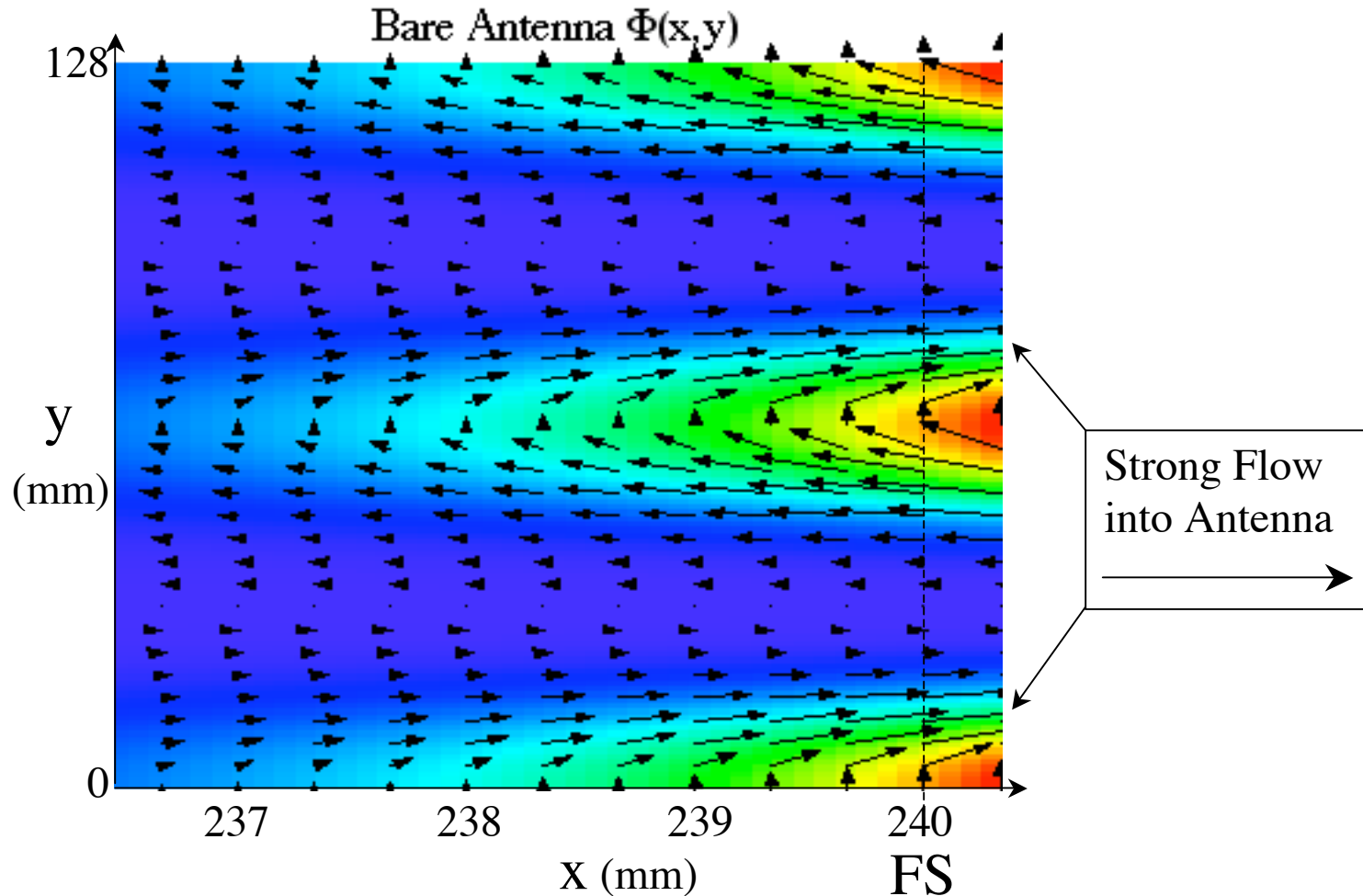
Benchmarking Exercise (3)

An initial sinusoidal perturbation of an exponential density profile develops a dipole vorticity that propels it into the SOL, consistent with simple theory ($T_e = \text{constant}$).



High RF Sheath Voltages \Rightarrow Strong Φ Near the Faraday Screen

- resulting strong flow can affect **blob propagation** and
- **pump plasma into the antenna**

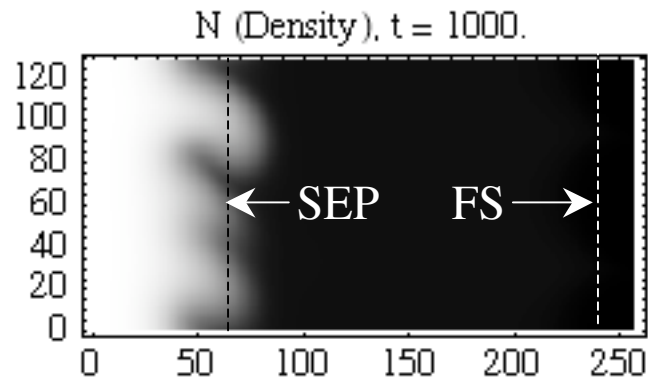
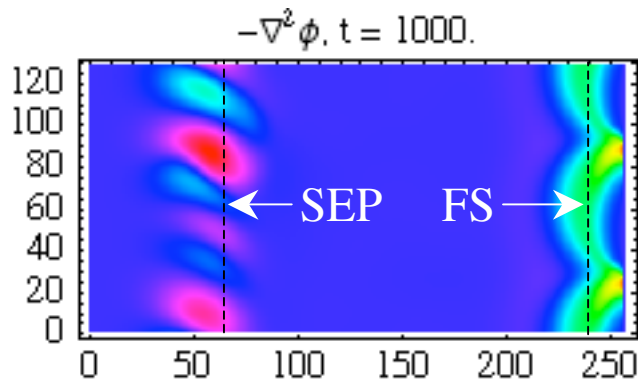


Fundamental Problem: How does antenna/plasma interaction mediate turbulence in the edge, if close “enough,” and in the SOL ?

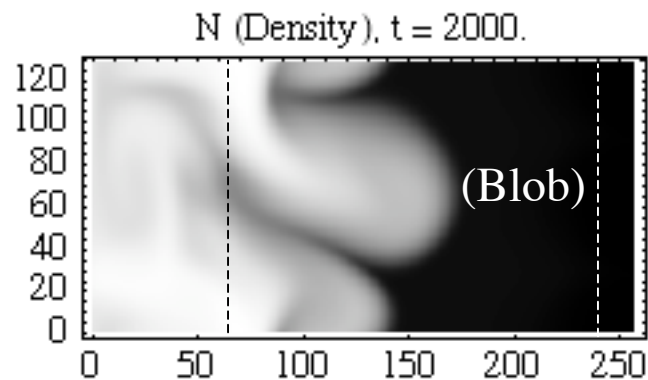
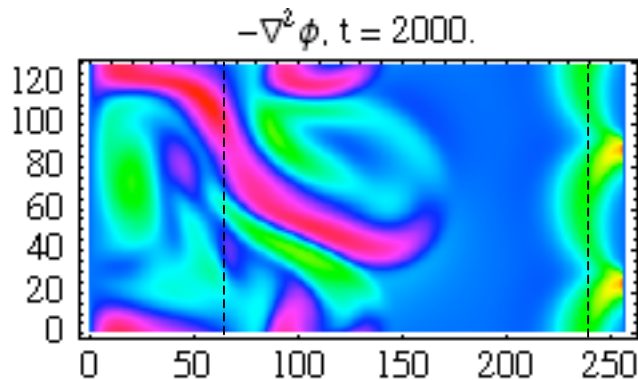
Important Application: How much damage will the antenna sustain due to this interaction?

Driven Turbulence ($T_e = \text{constant}$)

Instability at the Shoulder

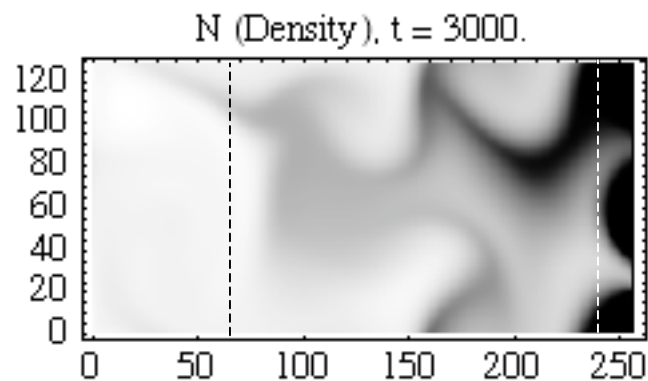
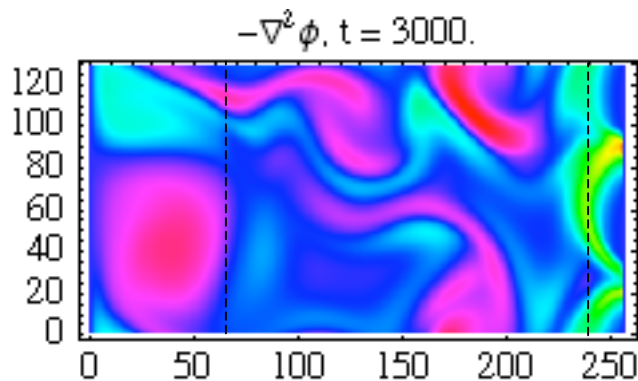


Blobs Propagating into the SOL

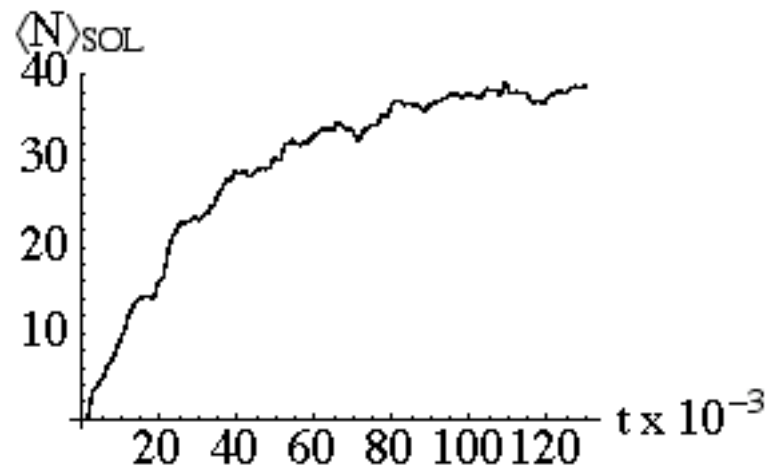
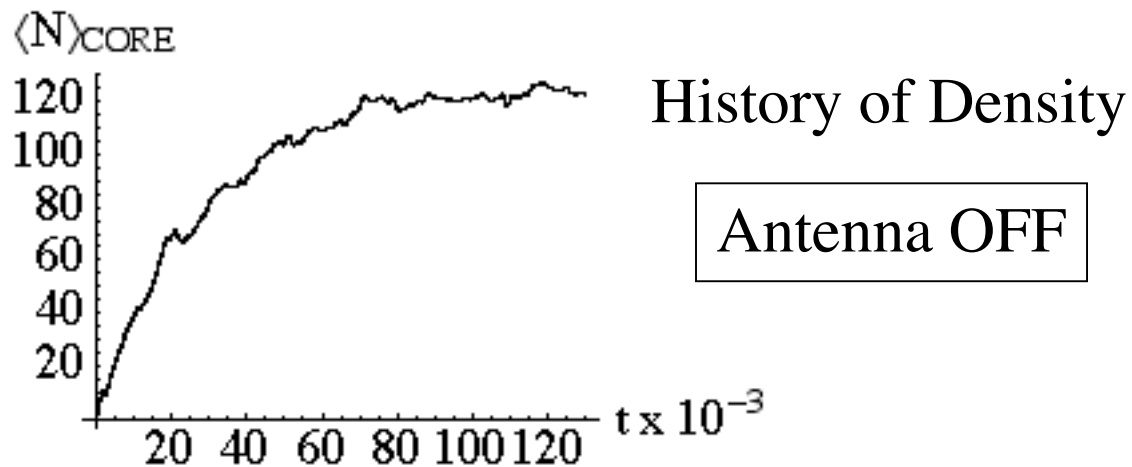


Strong Antenna Interaction

($V_a/T_e = 20$)

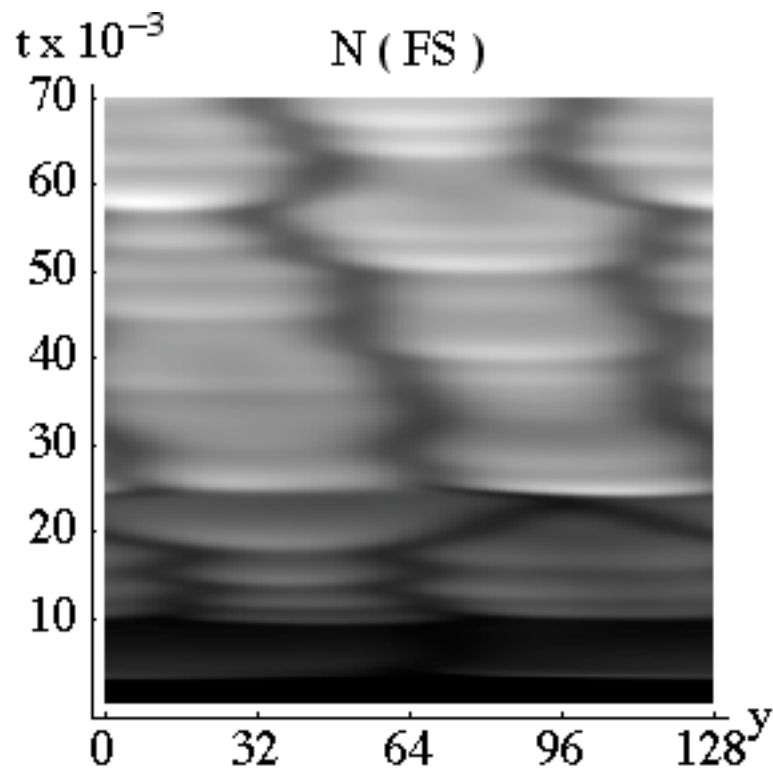
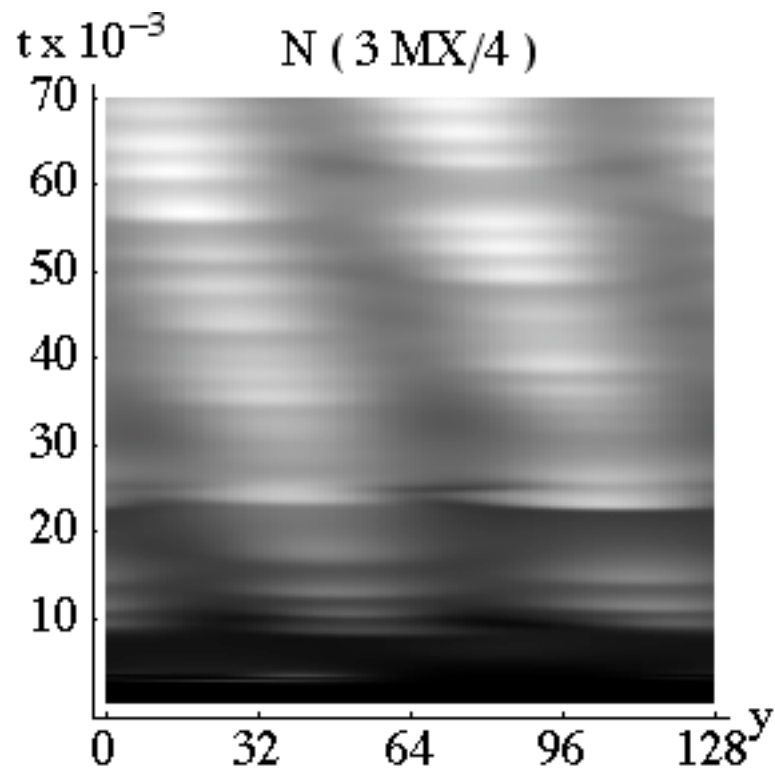


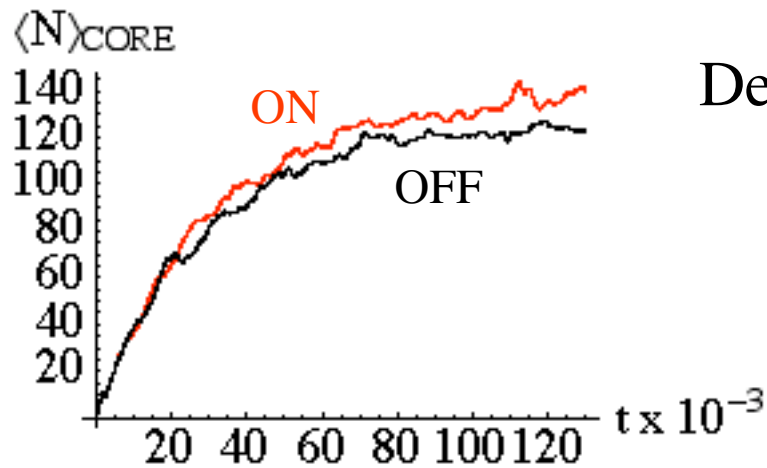
Log(N) is plotted.



Mid-SOL

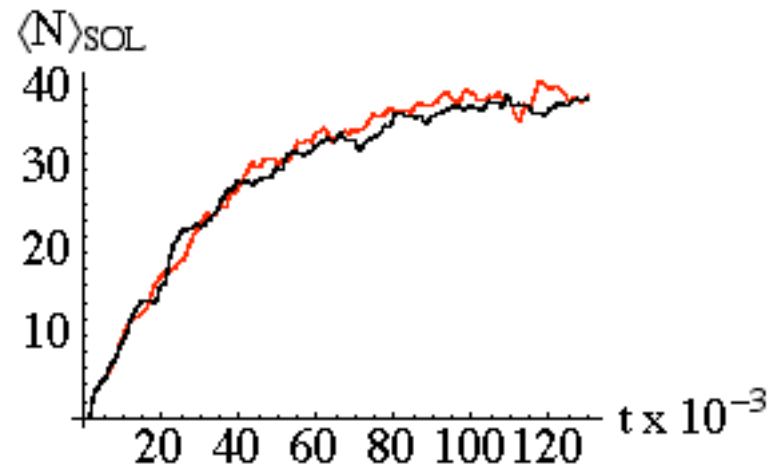
Faraday Screen



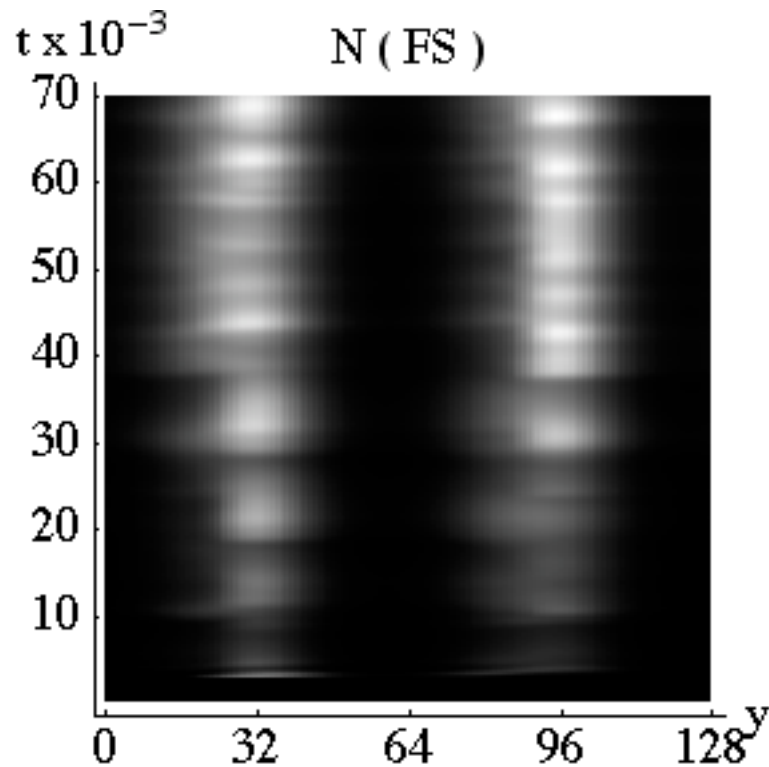
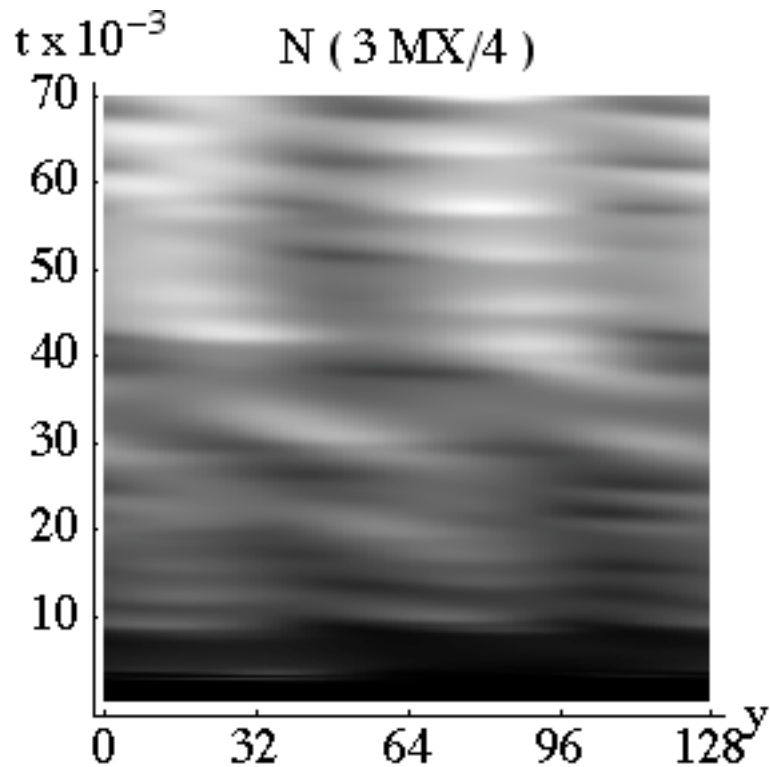


Density History

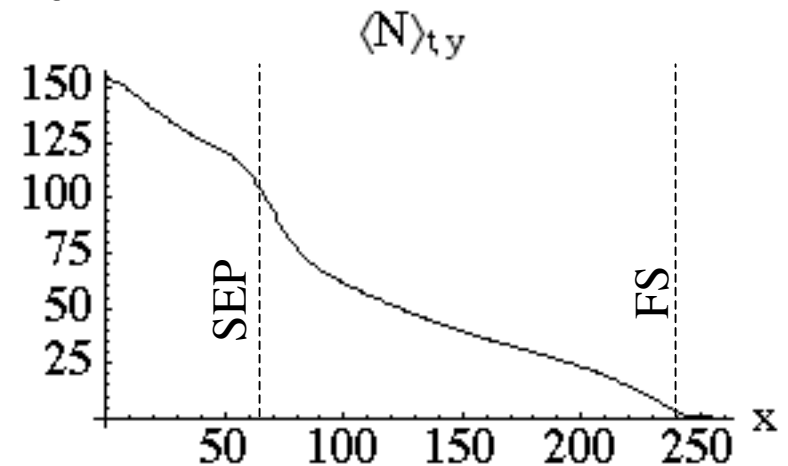
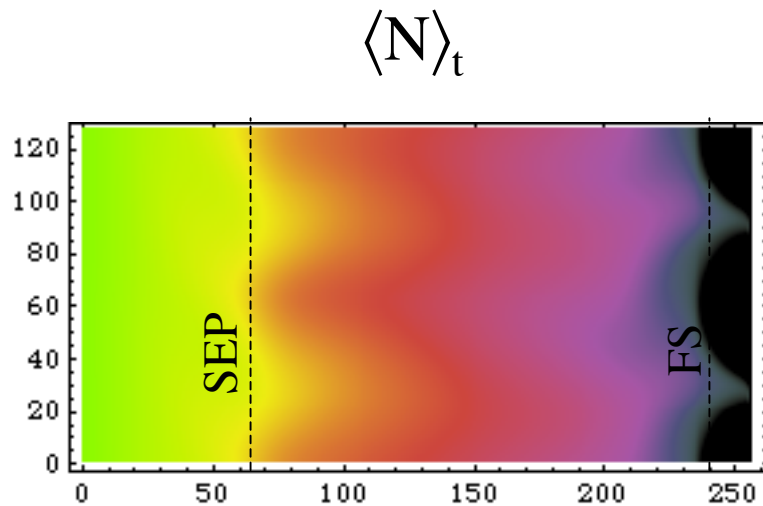
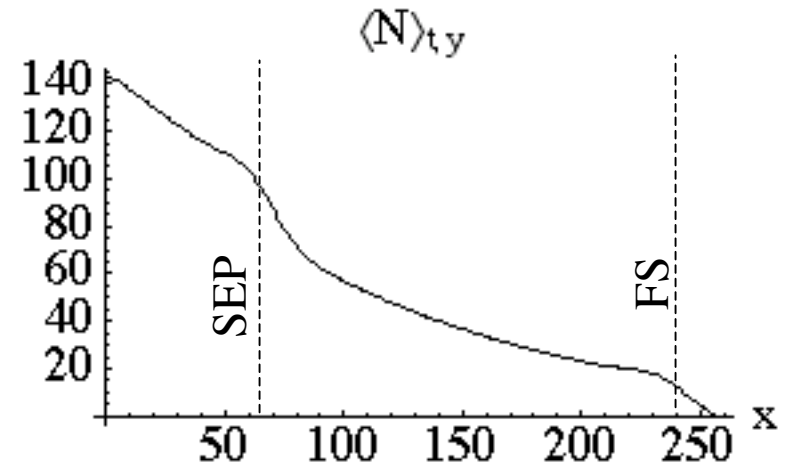
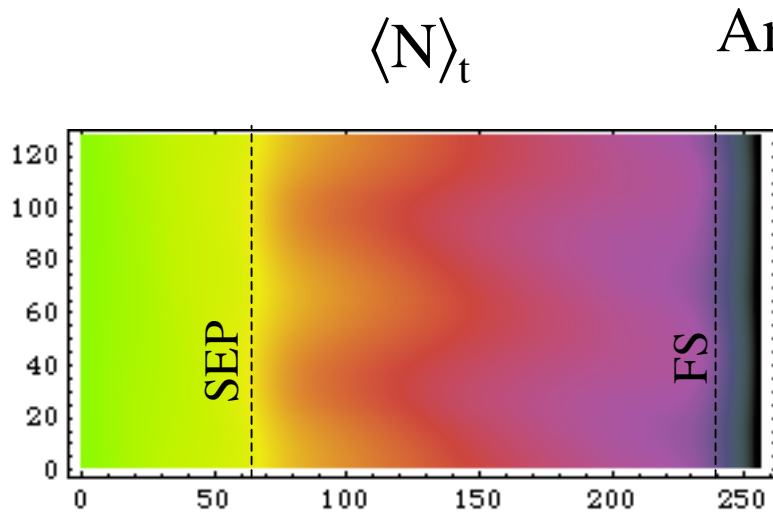
Antenna
 $V_a = 20$



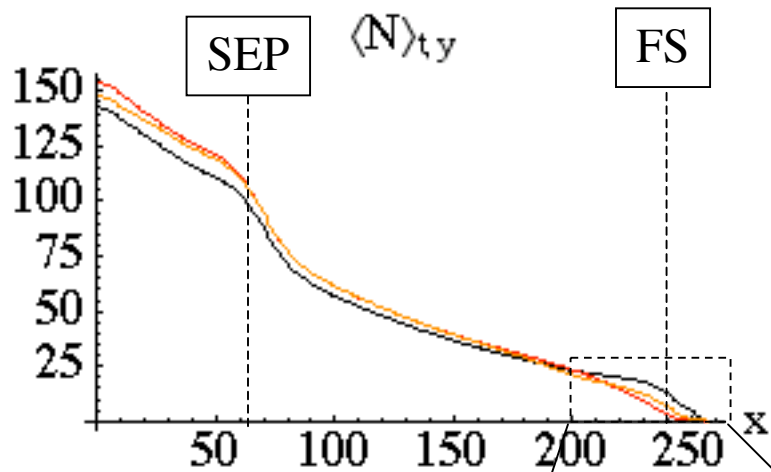
Notice the channeling of the flux near the FS,
 consistent with the antenna pattern.



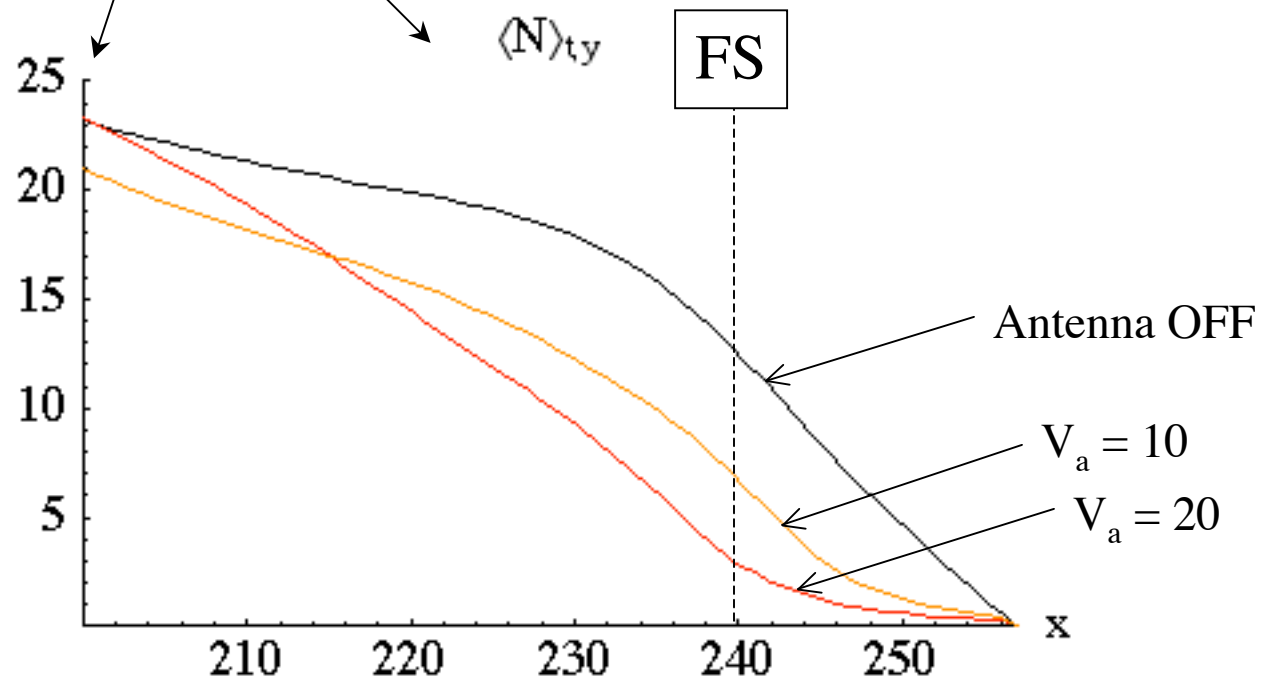
The Turbulent Density Profile falls off ~Linearly in the SOL.



The average is taken over the last half of the run.

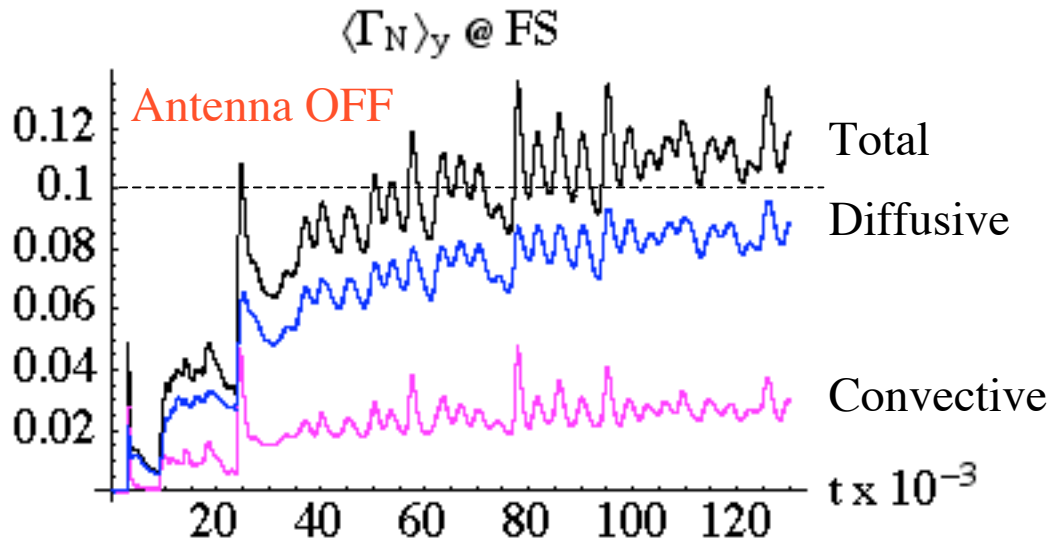


Stronger antenna potentials tend increasingly to evacuate the density in front of the Faraday screen and to flatten the profile there.

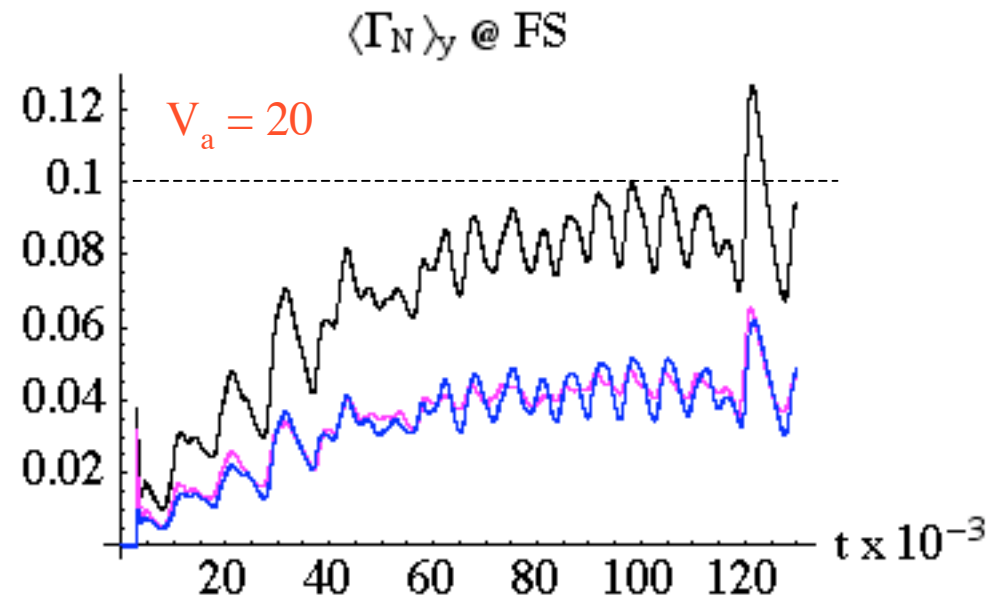
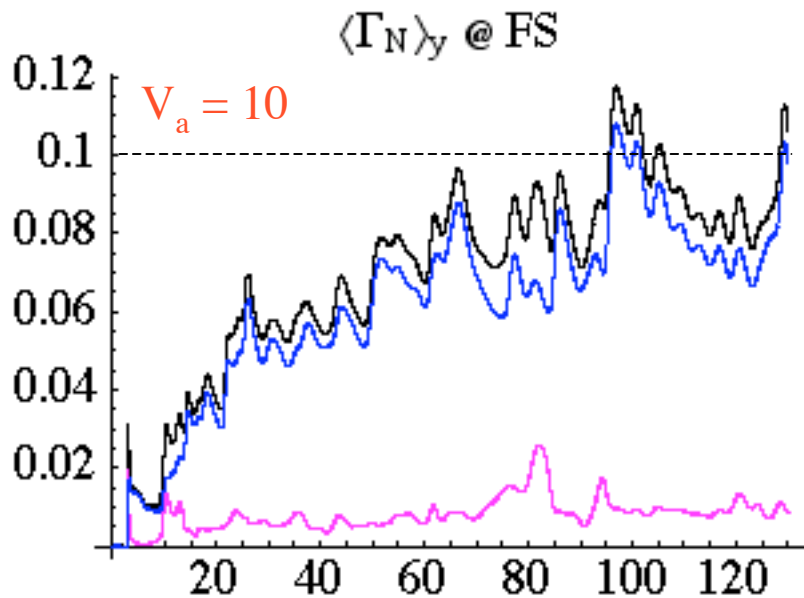


Particle Flux at the Faraday Screen

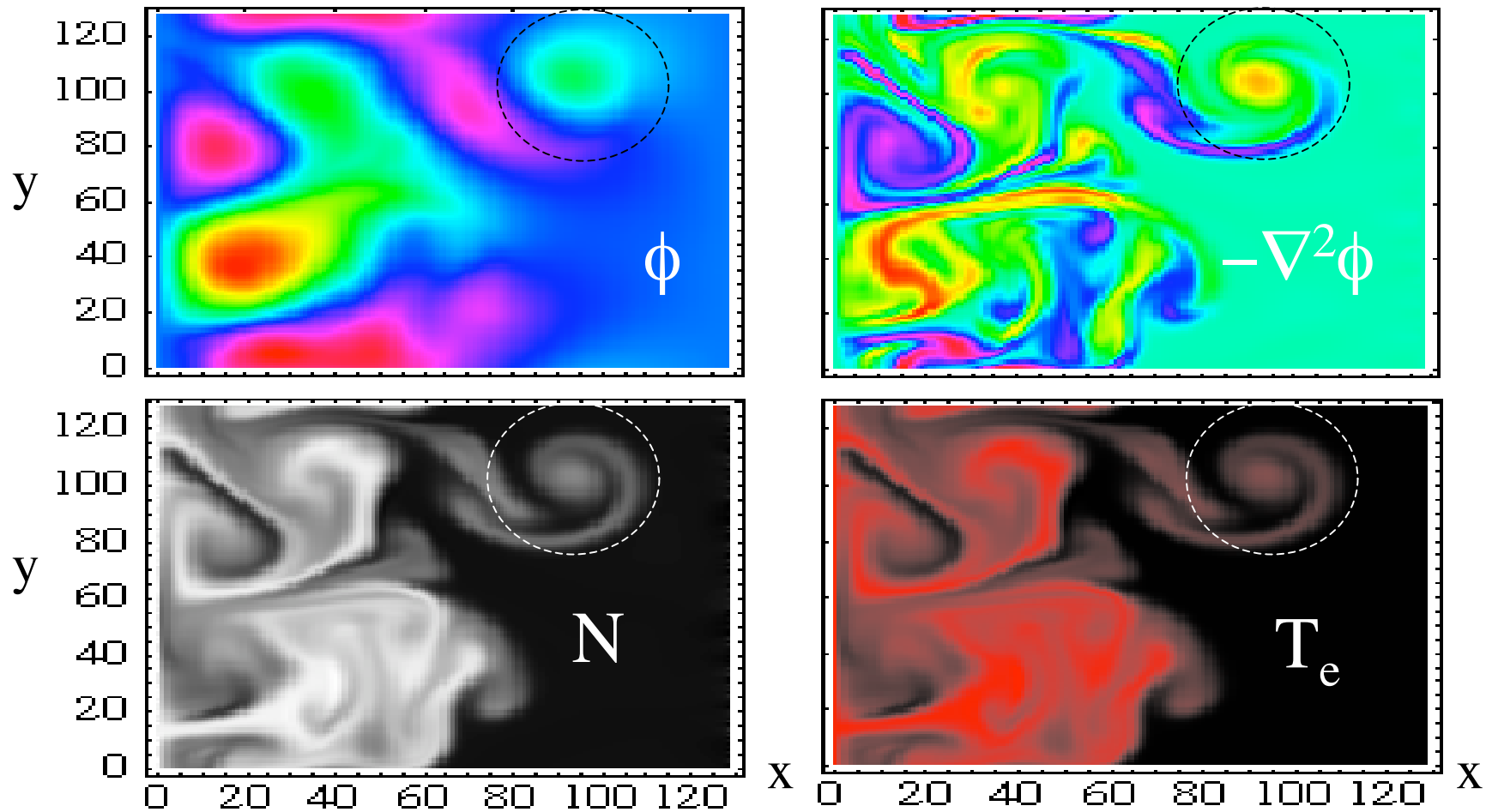
$$\Gamma_N = N \cdot V_x - D \cdot \partial_x N$$



- Why is the flux reduced with the antenna on? Stagnation?
- How is the flux partitioned between diffusion and convection?
- Are there antenna resonances?



T_e Evolved :



These are **spinning, monopole blobs**, $T_e \sim \phi$, as expected for **sheath boundary conditions**; they do not translate across the SOL as fast as their constant- T_e , dipole cousins and are more likely to drain away down the field lines before reaching the Faraday screen.

In Summary, we have demonstrated the feasibility of modeling rf antenna interaction with the edge and SOL plasma for cases where that interaction is mediated by strong (blob) turbulence.

We have

- Benchmarked a reduced-model 2D turbulence code and
 - Confirmed growth rates of the Nedospasov / R-T instability
 - Confirmed isolated blob structure and evolution for constant T_e
- Included rf-sheath terms and
 - Observed strong-turbulent emission of (dipole) blobs into the SOL for constant T_e
 - Observed anticipated spinning (monopole) blobs with T_e evolution
 - Observed strong antenna-plasma interaction including
 - Modulation of far SOL profiles and
 - Entrainment of blob trajectories

It remains to be seen if the simple 2D model can produce results consistent with experiment. To that end we envision modifying the sheath-connected boundary condition on $J_{||}$ with functional constraints on ϕ , N and T_e that capture the essential physics of sheath-*disconnected* turbulent transport in the SOL. We shall also study rf modifications of turbulence *inside* the separatrix.