Theory of Sheared Flow Generation by Applied Radio Frequency Waves

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Abstract. The possibility of employing rf to generate sheared flows in the edge plasma is of great interest as a means of accessing improved regimes of tokamak confinement. Here, we develop an electromagnetic nonlinear eikonal theory (with $k_\perp\rho \sim 1$ and $\mathbf{k}/k$ unrestricted) of the rf force terms which drive poloidal flow. Various cancellations, e.g. amongst parts of the electromagnetic and Reynolds stress terms, are exhibited analytically. At the heart of our calculation is the derivation of the nonlinear kinetic pressure tensor Π. A general expression for Π is obtained in terms of simple moments of the *linear* distribution function. In the electrostatic limit, the resulting nonlinear forces are expressible entirely in terms of the linear dielectric susceptibility tensor χ. Application to the ion Bernstein wave case, with retention of all Bessel function sums, is presented. Comparison is made to simpler approximate calculations.

INTRODUCTION

The possibility of employing rf to generate sheared flows in the edge plasma is of great interest as a means of accessing improved regimes of tokamak confinement. A body of theoretical work [1-6] has established possible mechanisms and scenarios, and serves as a plausible explanation of results that have been reported on a variety of experiments [7-11]. Such novel applications of ICRF waves are interesting because they could provide a degree of active external control over an internal transport barrier.

The basic idea [2] is that the poloidal component of force balance for the plasma is strongly affected by the rf through the nonlinearities (essentially the poloidal ponderomotive force F_y) giving rise to a net flow

$$V_y = F_y / \gamma_\theta n m_i, \tag{1}$$

where γ_θ is the neoclassical damping rate for poloidal flows. Note that because Eq. (1) derives from the fluid momentum equation, we require the force per unit volume on a *fluid* element, subtly different from the single particle guiding center ponderomotive forces for which a powerful formalism exists [12]. Thus an rf wave possessing the right properties can drive a sheared poloidal flow, $\partial V_y/\partial x$. The desired flow drive effects are particularly strong for the ion Bernstein wave (IBW) which has strong nonlinearities because it has a slow group velocity ($\partial\omega/\partial k \sim v_{ti}$) causing the wave amplitude to be large for a given transmitted power. Recent work [6] has incorporated sheared poloidal flow generation theory into full wave ICRF codes. Some limitations of the earlier calculations are discussed in Ref. 6. Many subtle cancellations have been found to occur when all of the nonlinear terms relevant to a given ICRF scenario are retained. In the present work, we develop a complementary formalism to that described in Ref. 6, based upon an eikonal expansion. Our formalism leads to analytical results which exhibit various cancellations explicitly, and which may therefore be a useful starting point for obtaining further physical insight into the underlying processes.

Excluded from discussion here are the radial ponderomotive force F_x which can generate poloidal flows through $\mathbf{F}\times\mathbf{B}$ (normally, this is a small effect), or the toroidal ponderomotive force which can drive toroidal flows.

PONDEROMOTIVE FORCE AND NONLINEAR PRESSURE TENSOR

We proceed directly from the species fluid momentum equation where the Lorentz force is $\mathbf{F}_L = Zen\mathbf{E} + \mathbf{J} \times \mathbf{B}/c$ and the total nonlinear ponderomotive force (PF) on a fluid element is $\mathbf{F} = \langle\mathbf{F}_L\rangle - \nabla\cdot\langle\Pi\rangle$. Here the brackets $\langle...\rangle$ are a quasilinear time average and $\Pi = m \int d^3v\, \mathbf{vv}\, f$ is the nonlinear pressure tensor, and represents the heart of our calculation. First, however, we rewrite $\mathbf{F}_L$ in a more convenient form.

Consider the term $\langle n_1\mathbf{E}_1\rangle$, where $\mathbf{E} \equiv \mathbf{E}_1$ is the applied wave. The linearized density is $n_1 = (-i/Ze\omega)\nabla\cdot\mathbf{J}_1$ thus this term can be expressed entirely in terms of $\mathbf{E}$ and the polarization vector $\mathbf{P} \equiv (4\pi i/\omega)\, \mathbf{J}_1 = \chi\cdot\mathbf{E}$ where χ is the species linear dielectric tensor. Moreover $\langle\mathbf{J}_1 \times \mathbf{B}_1\rangle$ can also be written in terms of $\mathbf{E}$ and $\mathbf{P}$ by using Maxwell's equations to write $\mathbf{B}_1$ in terms of $\mathbf{E}$. After some algebra which employs the vector identity $\nabla\cdot(\mathbf{JK}) = \mathbf{K}\nabla\cdot\mathbf{J} + \mathbf{J}\cdot\nabla\mathbf{K}$ with $\mathbf{K} = \mathbf{E}^*$ to eliminate $\nabla\cdot\mathbf{J}$ one can express the total PF as [13],

$$16\pi\, \mathbf{F} = [(\nabla\, \mathbf{E}^*)\cdot\mathbf{P} \;-\; \nabla\cdot(\mathbf{PE}^*) \;+ \text{cc}] - 16\pi\nabla\cdot\langle\Pi\rangle \tag{2}$$

In the fluid limit, where $\Pi = nm\mathbf{uu}$ with $\mathbf{u}$ the fluid velocity, Eq, (2) is equivalent to several other well-known forms for the PF [14]. When species summed, the PF is also equivalent to the divergence of the Maxwell stress-tensor. These latter forms, however, are not as convenient for present purposes as Eq. (2).

Proceeding with a kinetic evaluation of $\langle\Pi\rangle$, we split the contributions up according to their gyrophase (ϕ) dependence,

$$\begin{aligned}\langle\Pi\rangle &= m\int d^3v\, (\mathbf{vv} - \langle\mathbf{vv}\rangle_\phi)\, \tilde{f} \;+\; m\int d^3v\, \langle\mathbf{vv}\rangle_\phi \langle f\rangle_\phi \\ &= \Pi_{osc} \;+\; \Pi_{avg}\end{aligned} \tag{3}$$

where $\tilde{f}$ is the gyrophase dependent piece of the second order (quasilinear) distribution function $f = f_2$, and $\langle\rangle_\phi$ is a gyrophase average.

In the present paper, we focus on F_y due to radial (x) gradients of $|E|^2$; thus, the diagonal tensor Π_{avg} does not contribute, and a discussion of it will be deferred to a future publication. Here, we show how to write Π_{osc} in terms of $\mathbf{E}$ and moments of the linearized distribution function f_1, completing the implementation of Eq. (2).

To this end, we define the indefinite gyrophase integral $M = \int d\phi\, (\mathbf{vv} - \langle\mathbf{vv}\rangle)$ which can be expressed in terms of dyads involving $\mathbf{v}_\perp$, $\mathbf{v}_{||}$, and $\mathbf{b} = \mathbf{B}_0/B_0$. A parts integration of Eq. (3) casts the ϕ derivative from M onto f. Employing the quasilinear Vlasov equation for $\tilde{f}$, $\Omega\partial\tilde{f}/\partial\phi = \nabla_v\cdot\langle\mathbf{a}_1 f_1\rangle$, where the acceleration $\mathbf{a} = (Ze/m)\mathbf{E}_1 + (Ze/mc)\mathbf{v}\times\mathbf{B}_1$ and $\Omega = ZeB/mc$, we obtain after some algebra

$$\Pi_{osc} = \frac{m}{4\Omega}\int d^3v\, f_1\left[\frac{1}{4}(\mathbf{a}^*\mathbf{v}\times\mathbf{b} \;+\; \mathbf{v}\;\; \mathbf{a}^*\times\mathbf{b}) \;+\; \frac{3}{4}(\mathbf{a}_{||}{}^*\mathbf{v}\times\mathbf{b} \;+\; \mathbf{v}_{||}\;\; \mathbf{a}^*\times\mathbf{b}\;)\right] + \text{tr.} + \text{cc.} \tag{4}$$

where + tr. indicates the transpose of the preceding expression. Of particular interest is

$$\Pi_{xy} = \frac{m}{8\Omega}\int d^3v\; f_1\, (v_y a_y{}^* - v_x a_x{}^*) + \text{cc.} \tag{5}$$

Equation (4) is the desired general result. Two subsidiary limits are of interest. In the electrostatic limit $\mathbf{a} \to Ze\mathbf{E}/m$, and $\mathbf{a}$ may be pulled outside the velocity integral.

This is also the case in the electromagnetic fluid limit, where $kv/\omega << 1$ renders the $\mathbf{v}\times\mathbf{B}_1$ terms in **a** negligible. Thus, in either case we have

$$\Pi_{osc} = \frac{mn}{\Omega} <(\mathbf{a}_\perp\, \mathbf{u}\times\mathbf{b} + \mathbf{u}_\perp\, \mathbf{a}\times\mathbf{b})/4 \;+\; (\mathbf{a}_{||}\, \mathbf{u}\times\mathbf{b} + \mathbf{u}_{||}\, \mathbf{a}\times\mathbf{b}\,)> + \mathrm{tr.} \tag{6}$$

or for Π_{xy}, expressing the result in terms of the linearized current

$$\Pi_{xy} = \frac{1}{8\Omega}\,(J_y E_y{}^* - J_x E_x{}^*) + \mathrm{cc.} \tag{7}$$

In the fluid limit, where $\mathbf{a}_\perp = \partial\mathbf{u}_\perp/\partial t + \Omega\times\mathbf{u}$, it follows that $\Pi_{xy} = mn<u_x u_y>$ and the total PF of Eq. (2) vanishes, i.e. the Lorentz and fluid pressure tensor terms cancel identically. However, a nonzero result is possible when finite gyroradius corrections are important, as considered next.

APPLICATION TO THE IBW

For specificity, we consider the case of an electrostatic IBW with non-dissipative ions (i.e. sufficiently far from cyclotron resonance). The radial gradient required for a net force is taken to arise from other physics, e.g. electron dissipation. Under these conditions, χ is Hermitian, and Eq. (7) reduces to

$$\Pi_{xy} = -\,\frac{\omega\chi_{i\times} k_x k_y |\Phi|^2}{8\pi\Omega_i} \tag{8}$$

where Φ is the electrostatic potential and the components of χ are defined by $P_x = \chi_1 E_x + i\chi_\times E_y$, $P_y = \chi_2 E_y - i\chi_\times E_x$, and are well known in terms of standard Bessel function sums [15]. The net PF from Eq. (2) is the sum of the kinetic pressure and the Lorentz force terms,

$$F_y = \frac{1}{8\pi}\, k_x k_y \partial_x |\Phi|^2 \left(\frac{\omega}{\Omega_i}\,\chi_{i\times} - \chi_{i1}\right). \tag{9}$$

We note that $k_x k_y \neq 0$ is a necessary condition for a non-vanishing poloidal force in the electrostatic limit considered here.

It is interesting to compare the result of our complete kinetic calculation, Eq. (8) to an *ad hoc* generalization of the fluid result $\Pi_{xy} = mn<u_x u_y> = m/(Z^2e^2n)<J_x J_y>$ obtained by employing the kinetic Bessel expressions for J_x and J_y. This procedure leads to

$$\hat{\Pi}_{xy} = \frac{\omega^2(\chi_{i1}{}^2 - \chi_{i\times}{}^2) k_x k_y |\Phi|^2}{8\pi\omega_{pi}{}^2}. \tag{10}$$

Shown in Fig. 1 for the case $\omega/\Omega_i = 2.1$, is the total normalized PF driving sheared flow, the individual contributions of the kinetic pressure and the Lorentz force terms, and the PF as calculated using the *ad hoc* pressure tensor of Eq. (10). Note the cancellation of Lorentz and kinetic pressure terms for $k_\perp\rho_i \to 0$. The *ad hoc* fluid result is qualitatively valid for $k_\perp\rho_i << 1$, tends to be an overestimate for $k_\perp\rho_i \sim 1$ and $\omega \to n\Omega_i$, and can have the wrong sign. Note from Eq. (9) that the sign of F_y is odd in $\omega - n\Omega_i$ due to the linear dependence on χ.

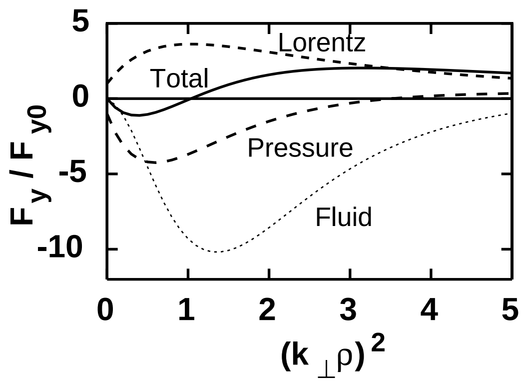

Figure 1. Total normalized F_y (solid) and individual terms: Lorentz (short dashed) and kinetic pressure (long dashed). Also shown is the total fluid F_y obtained using Eq. (10). F_y is normalized to F_{y0}, the Lorentz term for $k_\perp\rho = 0$.

CONCLUSIONS

A systematic formulation of the ponderomotive force on a fluid element due to electromagnetic waves in hot kinetic plasmas describable within the eikonal approximation has been presented. The results are expressible in terms of linearized moments of the perturbed distribution function, viz. the linearized current density and pressure. Cancellations between the Lorentz force terms and the quasilinear kinetic pressure term have been exhibited analytically. Numerical calculations, shown in Fig. 1, indicate that a correct representation of the kinetic pressure is important to describe the rf forcing of sheared flows for IBWs with $k_\perp\rho_i \gtrsim 0.2$. These results are in qualitative agreement with the full wave, $k_\perp\rho_i < 1$ numerical results of Ref. 6.

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