Poloidal force generation by applied radio frequency waves

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A theoretical framework is developed for calculating the nonlinear rf forces that can drive sheared poloidal flow in a tokamak plasma. It is shown that the rf-induced flow drive can be calculated without first obtaining an explicit result for the nonlinear distribution function. Instead, for modes satisfying the eikonal approximation, the flow drive can be expressed entirely in terms of moments of the linearized plasma responses. The method is applied to obtain explicit results for poloidal force generation for sheared flow drive applications in a hot plasma slab which supports rf waves of arbitrary polarization. The theory is fully electromagnetic and retains $k_\perp \rho_i \sim 1$ (Bessel function) effects for the ion dynamics without approximation. An illustrative application to the ion Bernstein wave is presented.

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I. Introduction

Recently, there has been considerable interest in the possibility of employing rf waves, particularly in the ion cyclotron range of frequencies (ICRF), to induce shear in the poloidal flow velocity of tokamak plasmas. Theory[1-7] suggests that rf-induced sheared flows may allow access to high confinement regimes and thus be a key to greatly improved tokamak performance. Confinement improvement as a result of the application of rf power has been reported on a numbers of experiments,[8-12] and at least in some cases, rf-induced sheared flows provide a plausible explanation of the observations. Such novel applications of ICRF waves could provide a degree of active external control over an internal transport barrier, that could be important for advanced tokamaks.

A core H-mode induced by the application of ion Bernstein wave (IBW) power has been demonstrated on the Princeton Beta Experiment Modifications (PBX-M).[10] ICRF was successfully employed on a deuterium-tritium (D-T) plasma in the Tokamak Fusion Test Reactor (TFTR) to create a poloidally shear flow layer that could be directly diagnosed.[11, 12] Experiments related to shear flow generation and confinement improvement are also in progress on the Frascati Tokamak Upgrade (FTU)[13] and are planned on the Alcator C-Mod tokamak.[14]

There have been several attempts in the past to describe the nonlinear theory of rf interactions with the plasma and the generation of sheared flows.[1-7] Recently, Berry *et al*.[6,7] have developed a kinetic theory of rf-induced shear flow generation. Their theory was implemented in an ICRF code and subsequently employed to assess the prospects of ICRF shear flow generation in present day experiments, and to compare results of their kinetic derivation with earlier work in which a number of approximations were made.

The complexity of previous calculations motivates a search for an efficient derivation which can lead to relatively compact and transparent results. This is the goal of the present paper in which we exploit the eikonal formalism in an attempt to obtain simplified results which might be useful for understanding the underlying physical mechanisms of poloidal force generation. Our paper supplements and extends an earlier conference paper[15] in which some useful, labor-saving calculational methods were established.

The model considered here is a local slab model for an rf wave satisfying the eikonal approximation. However, the present results can also be applied locally to the individual modes in a spectral decomposition of arbitrary wave fields, as might be obtainable from within a full wave code.

The basic idea[1,2] underlying sheared flow drive is that the poloidal (y) component of the force balance equation in a tokamak contains only small terms, making the plasma susceptible to poloidal flows by the application of relatively small poloidal forces. For present purposes, we consider that the nonlinear rf-induced force F_y must balance an effective neoclassical damping, so that the resulting flow velocity V_y is

$$V_y = F_y / \gamma_\theta n m_i, \qquad (1)$$

where γ_θ is the neoclassical damping rate, n is the plasma density and m_i is ion mass. Note that because Eq. (1) derives from the fluid momentum equation, we require the force per unit volume on a *fluid* element, subtly different from the single particle guiding center ponderomotive forces for which a powerful formalism exists.[16]

Thus, an rf wave possessing the right properties can drive a sheared poloidal flow, $\partial V_y/\partial x$. The desired flow drive effects are particularly strong for the IBW which has strong nonlinearities because it has a slow group velocity ($\partial\omega/\partial k \sim v_{ti}$) causing the wave amplitude to be large for a given transmitted power. Consequently, it is important to retain $k_\perp\rho_i \sim 1$ effects in the derivations (i.e. full Bessel functions). Here $k_\perp$ is the wavevector perpendicular to the equilibrium magnetic field **B**, and ρ_i is the ion Larmor radius.

The ultimate goal of this work is stabilization or suppression of turbulence. The present paper treats only one aspect of the calculation - the poloidal force on the equilibrium plasma applied by rf waves. Other considerations, outside the scope of the present paper, are the redistribution of charges due to the rf, which could create a radial electric field E_r and concomitant E×B poloidal rotation, and neoclassical effects which in principle should be included self-consistently in the quasilinear (zero frequency) beat response of the plasma to the rf waves. Additionally, the stability of low frequency (relative to the rf) turbulent modes in the presence of rf may not be fully describable by including rf-induced equilibrium modifications alone. Three wave interactions and wave-particle interactions involving the rf and the low frequency mode could also enter. Without dismissing the importance of any of these effects, the present calculation, while limited in scope, represents one conceptual step.

The plan of our paper is as follows. In Sec. II we recapitulate the basic formalism developed in Ref. 15 as applicable to the present paper. In Sec. III we obtain explicit electromagnetic results for the nonlinear force, extending the electrostatic limit previously considered.[15] The calculation is applicable to any wave in a magnetized collisionless plasma for which the eikonal, spectral width [Eq. (22)] and guiding center [Eq. (23)] approximations apply. While the authors are primarily motivated by ICRF applications, the theory is not explicitly restricted in rf frequency. In Sec. IV we consider two simple applications to the IBW, which illustrate the possibility of cancellations, and the role of finite Larmor radius and dissipative effects. Further discussion of the physical requirements for an rf-induced poloidal force by the present mechanism is given in Sec. V. Finally, Sec. VI gives the conclusions. In Appendix A the relationship of present results to various forms of the ponderomotive force in the literature is summarized. For completeness, diagonal terms of the nonlinear pressure tensor, unimportant for sheared flow drive, are briefly considered in Appendix B. Appendix C presents some tedious details necessary to obtain the complete electromagnetic flow drive results of Sec. III.

II. Ponderomotive Force and Nonlinear Pressure Tensor

A. Basic Formalism

In this section the formalism for calculating the total nonlinear time-averaged second order force on the equilibrium due to the rf waves is presented. We will refer to this force as the ponderomotive force. It is a generalization of the usual ponderomotive force on a fluid element, in that it includes dissipative and finite Larmor radius effects.

For each species, the continuity and momentum equations may be written as

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(nm\mathbf{u}) + \nabla\cdot\Pi = \mathbf{F}_L \equiv Zen\mathbf{E} + \frac{1}{c}\mathbf{J}\times\mathbf{B} \tag{3}$$

where n, $\mathbf{u}$, m, Ze, and $\mathbf{J} = Zen\mathbf{u}$ are the species density, flow velocity, mass, charge and current density respectively, $\mathbf{F}_L$ is the Lorentz force, and Π is the species pressure tensor. The quantities n, $\mathbf{u}$ and Π are defined, as usual, in terms of the particle distribution function f by

$$n = \int d^3v\ f, \tag{4}$$

$$n\mathbf{u} = \int d^3v\ \mathbf{v}\ f, \tag{5}$$

$$\Pi = m\int d^3v\ \mathbf{v}\mathbf{v}\ f, \tag{6}$$

where $\mathbf{v}$ is the particle velocity in the reference frame. It is important to emphasize that Eqs. (2) - (6) are valid even when fluid theory is not (e.g. $k_\perp\rho_i \sim 1$) provided that the correct kinetic expression for Π is employed.

When Eqs. (2) and (3) are linearized with respect to the rf field contributions in $\mathbf{E}$ and $\mathbf{B}$, and combined with Maxwell's equations, expressions for the linearized conductivity tensor σ may be obtained (again assuming Π is known). Time averaging Eqs. (2) and (3) and working in second order in the rf field amplitude yields the nonlinear rf forces acting on the equilibrium. Taking the equilibrium $\mathbf{u}$ and $\mathbf{E}$ to be zero in the reference frame, the ponderomotive force (PF) density (i.e. the total nonlinear force per unit volume) from Eq. (3) is

$$\mathbf{F} = <\mathbf{F}_L> - \nabla\cdot<\Pi>, \tag{7}$$

and consists of contributions from the nonlinear Lorentz force and from the nonlinear kinetic pressure tensor. Here <...> represents a quasilinear time average which picks out the zero frequency beat terms.

Examining the Lorentz force terms first, from the perturbed continuity equation, we obtain

$$n_1 = \frac{-i}{Ze\omega}\nabla\cdot\mathbf{J} \tag{8}$$

where subscript 1 denotes linearized quantities, but for notational convenience the 1 will be dropped on **J**, **u** and **E** unless the meaning is ambiguous. Using Maxwell's equations to write $\mathbf{B}_1$ in terms of **E**, and noting that

$$<AB> = (1/4)AB^* + cc, \tag{9}$$

the Lorentz force contribution may be written as

$$<\mathbf{F}_L> = \frac{i}{4\omega}[-\mathbf{E}^*\,\nabla\cdot\mathbf{J} + \mathbf{J}\times(\nabla\times\mathbf{E}^*)] + cc$$

$$= \frac{i}{4\omega}[-\mathbf{E}^*\,\nabla\cdot\mathbf{J} + (\nabla\,\mathbf{E}^*)\cdot\mathbf{J} - \mathbf{J}\cdot\nabla\mathbf{E}^*] + cc \tag{10}$$

Then using $\nabla\cdot(\mathbf{JK}) = \mathbf{K}\nabla\cdot\mathbf{J} + \mathbf{J}\cdot\nabla\mathbf{K}$ with $\mathbf{K} = \mathbf{E}^*$ to eliminate the $\nabla\cdot\mathbf{J}$ term yields

$$16\pi\,\mathbf{F}_L = (\nabla\,\mathbf{E}^*)\cdot\mathbf{P} - \nabla\cdot(\mathbf{PE}^*) + cc \tag{11}$$

where henceforth we use the notation $\mathbf{F}_L$ for $<\mathbf{F}_L>$. Here the polarization vector

$$\mathbf{P} \equiv \chi\cdot\mathbf{E} \tag{12}$$

has been introduced, where

$$\chi \equiv \frac{4\pi i}{\omega}\sigma \tag{13}$$

is the species dielectric susceptibility tensor and σ is the conductivity tensor ($\mathbf{J} = \sigma$E). Thus the total PF can be written as

$$\mathbf{F} = \frac{1}{16\pi}[(\nabla\,\mathbf{E}^*)\cdot\mathbf{P} - \nabla\cdot(\mathbf{PE}^*) + cc] - \nabla\cdot<\Pi> \tag{14}$$

Equation (14) is the main result of this subsection. It expresses the part of the PF in terms of linearized field quantities which can be related to the usual linear conductivity.

The relationship of Eq. (14) to other forms of the PF that have appeared in the literature is considered in Appendix A.

B. Off-diagonal terms of the stress tensor

1. General form

We now proceed with a kinetic calculation of $\langle\Pi\rangle$, which is the heart of our paper. The goal is obtain an expression for the nonlinear second order $\langle\Pi\rangle$ in terms of moments of the linearized (first order) distribution function. To begin, we split the contributions up according to their gyrophase (ϕ) dependence,

$$\langle\Pi\rangle = m\int d^3v\ (\mathbf{vv} - \langle\mathbf{vv}\rangle_\phi)\ \tilde{f} + m\int d^3v\ \langle\mathbf{vv}\rangle_\phi \langle f\rangle_\phi \qquad (15)$$

$$= \Pi_{osc} + \Pi_{avg}$$

where $\tilde{f} = f - \langle f\rangle_\phi$ is the gyrophase dependent piece and $\langle f\rangle_\phi$ is the gyroaveraged piece of the quasilinear distribution function $f = f_2$, and $\langle\rangle_\phi = \int d\phi/2\pi$ is a gyrophase average.

For the present application, the calculation of sheared flow, we require the poloidal force F_y resulting from radial (∂_x) gradients in the rf field amplitude. Thus, only the off-diagonal terms Π_{xy} are required. These are contained in Π_{osc} and arise from $\tilde{f}$. It is significant that $\langle f\rangle_\phi$ is not required, because $\langle f\rangle_\phi$ is actually considerably more difficult to calculate than $\tilde{f}$. The diagonal tensor Π_{avg}, which does depend on $\langle f\rangle_\phi$, can be obtained in some limiting cases, and these are discussed in Appendix B. Here we proceed with a general evaluation of Π_{osc}.

The basic idea is to write $\mathbf{vv} - \langle\mathbf{vv}\rangle$ as the derivative with respect to gyrophase ϕ of some quantity, and then do a parts integration to cast the ϕ derivative onto f_2. The Vlasov equation will then be employed to write $\partial f_2/\partial\phi$ in terms of the linearized distribution f_1.

In gyrokinetic variables

$$\mathbf{R} = \mathbf{r} + \mathbf{v}\times\mathbf{b}/\Omega \qquad (16)$$

$$\mathbf{v}_\perp = v_\perp\ (\mathbf{e}_x \cos\phi + \mathbf{e}_y \sin\phi) \qquad (17)$$

with $\mathbf{e}_x\times\mathbf{e}_y=\mathbf{e}_z=\mathbf{b}$ the Vlasov equation is

$$\frac{\partial f}{\partial t} + v_{||}\nabla_{||}f - \Omega\frac{\partial f}{\partial\phi} = -\mathbf{a}\cdot\nabla_v f \qquad (18)$$

where

$$\mathbf{a} = (Ze/m)\mathbf{E}_1 + (Ze/mc)\mathbf{v}\times\mathbf{B}_1 \qquad (19)$$

$$= \frac{Ze}{m} [I(1 + \mathbf{k}\cdot\mathbf{v}/\omega) - \mathbf{kv}/\omega]\cdot\mathbf{E}_1 \qquad (20)$$

is the acceleration due to all forces except the force in the zero order magnetic field. For the gyrophase dependent part of f_2 we have (dropping the ~ for notational brevity)

$$\Omega\frac{\partial f_2}{\partial\phi} = \nabla_v\cdot <\mathbf{a}_1 f_1>_t = \frac{1}{4}\nabla_v\cdot(\mathbf{a}_1{}^* f_1) + cc \qquad (21)$$

where we have used the fact that f_2 is driven by the zero frequency beat of f_1 with a_1 and is therefore independent of space and time to leading order.

The approximation of neglecting the spatial dependence of the zero-frequency beat $\mathbf{a}_1{}^* f_1$ in f_2 is the central approximation of our paper. Its importance in terms of simplification of results and also in the neglect of certain physical effects requires comment. The simplification results because the Vlasov propagation operator d/dt reduces to $\Omega\, \partial/\partial\phi$ which is easily invertible. The approximation is justifiable when the spectral width δk in k-space of E_1 obeys

$$\delta k\ \rho_i << 1, \qquad (22)$$

assuming also

$$\rho_i / L << 1 \qquad (23)$$

where $\rho_i = v/L$ may be estimated using the thermal velocity. These approximations are satisfied for a single k mode, describable within the framework of eikonal theory and the local approximation. In practice, the spatially localized resonant cyclotron layers important for ICRF and IBW damping may sometimes lead to the violation of these approximations. In these cases, retaining finite spectral width with $\delta k\ v / \Omega \sim 1$ becomes important. Spectral width issues will be the subject of a future publication. Here, we note that they are related to the subject of non-locality in energy absorption due to finite Larmor radius effects, as discussed by Smithe,[17] and more recently by Jaeger *et al.* [7]

Resuming the derivation, we next define the indefinite gyrophase integral

$$M = \int d\phi\ (\mathbf{vv} - <\mathbf{vv}>) = \frac{1}{4}(\mathbf{vv}\times\mathbf{b} + \mathbf{v}\times\mathbf{bv}) + \frac{3}{4}(\mathbf{v}_{||}\mathbf{v}\times\mathbf{b} + \mathbf{v}\times\mathbf{bv}_{||}) \qquad (24)$$

so that

$$\Pi_{osc} = -m\int d^3v\ M\,\frac{\partial f_2}{\partial\phi} = -\frac{m}{4\Omega}\int d^3v\ M\nabla_v\cdot(\mathbf{a}_1{}^* f_1) + cc$$

$$= \frac{m}{4\Omega}\int d^3v\ f_1\mathbf{a}_1{}^*\cdot\nabla_v M + cc. \qquad (25)$$

After some algebra, the last form can be written as

$$\Pi_{osc} = \frac{m}{4\Omega}\int d^3v\, f_1\left[\frac{1}{4}(\mathbf{a}^*\mathbf{v}\times\mathbf{b} + \mathbf{v}\ \mathbf{a}^*\times\mathbf{b}) + \frac{3}{4}(\mathbf{a}_{||}{}^*\mathbf{v}\times\mathbf{b} + \mathbf{v}_{||}\ \mathbf{a}^*\times\mathbf{b})\right]$$

$$+ \text{tr.} + \text{cc.} \tag{26}$$

where + tr. indicates that the transpose of the preceding expression is to be added to the result that is explicitly displayed.

Equation (26) is the desired general result. It expresses Π_{osc} completely in terms of moments of the linearized distribution function f_1. Two subsidiary limits are of interest and are discussed next.

2. Subsidiary limits

In the electrostatic limit the velocity dependent parts of **a** cancel, and **a** may be pulled outside the velocity integral. This is also the case in the electromagnetic fluid limit, where kv/ω << 1 is assumed. In either case we have

$$\Pi_{osc} = \frac{mn}{\Omega} <(\mathbf{a}_\perp\, \mathbf{u}\times\mathbf{b} + \mathbf{u}_\perp\, \mathbf{a}\times\mathbf{b})/4 + (\mathbf{a}_{||}\, \mathbf{u}\times\mathbf{b} + \mathbf{u}_{||}\, \mathbf{a}\times\mathbf{b})> + \text{tr.} \tag{27}$$

For later demonstration (Appendix B) that the correct result is obtained in the fluid limit, we can invoke the fluid accelerations

$$\mathbf{a} \rightarrow \frac{\partial \mathbf{u}}{\partial t} + \Omega\mathbf{b}\times\mathbf{u} \tag{28}$$

to obtain

$$\Pi_{osc}{}^{fluid} = \frac{mn}{2} <\mathbf{u}_\perp\, \mathbf{u}_\perp - \mathbf{u}\times\mathbf{b}\ \mathbf{u}\times\mathbf{b}> + mn <\mathbf{u}_{||}\mathbf{u}_\perp + \mathbf{u}_\perp\mathbf{u}_{||}>$$

$$= mn <\mathbf{u}\,\mathbf{u}> - \frac{mn}{2} <u_\perp{}^2> I_\perp - mn<u_{||}{}^2>\mathbf{bb} \tag{29}$$

where $u_\perp{}^2 I_\perp = \mathbf{u}_\perp\mathbf{u}_\perp + \mathbf{u}\times\mathbf{b}\ \mathbf{u}\times\mathbf{b}$ defines the perpendicular projection of the identity tensor.

III. Explicit Results for the Poloidal Force

Returning to the general electromagnetic result, Eq. (26), we now specialize to a pseudo-Cartesian plasma slab with x, y, z directions corresponding to radial, "poloidal" and parallel directions. Strictly speaking, the y coordinate points in the direction normal to both **b** and ∇ψ, and is thus not purely poloidal. Equation (26) for Π yields the desired sheared flow drive component

$$\Pi_{xy} = \frac{m}{8\Omega} \int d^3v\, f_1 \,(v_y a_y^* - v_x a_x^*) + cc. \tag{30}$$

Recall that the off-diagonal components of Π_{avg} are zero, so that the xy component of Π_{osc} given here is the complete off-diagonal result.

Employing Eq. (19) for **a**, Π_{xy} splits into contributions arising from the $\mathbf{E}_1$ and $\mathbf{B}_1$ pieces. It is readily seen that the integrals involving the $\mathbf{E}_1$ terms can be expressed in terms of the plasma current **J** while the integrals involving the $\mathbf{B}_1$ terms involve the linearized pressure tensor p_{ij} (i, j = x, y, z), given by

$$p_{ij} = \int d^3v\; f_1 v_i v_j \tag{31}$$

We are finally in a position to combine the force and pressure tensor contributions. Up to this point, the theory has been worked out assuming only Eqs. (22) and (23) together with the usual eikonal condition $k_\perp L >> 1$. For the explicit results quoted below, we further specialize to the case $k_y = 0$.

The total force in the y direction is, from Eq. (14),

$$F_y = -\frac{\partial}{\partial x} \,(U_L + U_E + U_B) \tag{32}$$

where the terms driving flow, U_L, U_E and U_B, represent respectively the Lorentz force, and contributions of $\mathbf{E}_1$ and $\mathbf{B}_1$ to the kinetic pressure tensor. Note that the first term in Eq. (14), $(k_y/\omega) \langle \mathbf{J}\cdot\mathbf{E} \rangle$ corresponds to the wave momentum absorbed by the plasma, which vanishes when $k_y = 0$. The remaining terms, Eq. (32), are explicitly momentum conserving, when integrated over the entire plasma volume. Thus, there is no net force F_y on the plasma in the limit $k_y = 0$, but there can be momentum redistribution which drives sheared flow through Eq. (1).

The contributions to Eq. (32) are summarized as follows:

$$U_L = \frac{i}{4\omega} J_x E_y^* + cc. \tag{33}$$

$$U_E = \frac{1}{8\Omega} (J_y E_y^* - J_x E_x^*) + cc. \tag{34}$$

$$U_B = -\frac{Ze}{8\Omega c} (2p_{xy} B_z^* + p_{yz} B_x^* + p_{xz} B_y^*) + cc. \tag{35}$$

where the components of **J** are obtained from $\mathbf{J} = \sigma\mathbf{E}$ and the p_{ij} are each expressed as linear combinations of the components of **E**, e.g.

$$p_{xy} = p_{xyx}\, E_x + p_{xyy}\, E_y + p_{xyz}\, E_z. \tag{36}$$

The σ_{ij} and p_{ijk} tensor elements are given explicitly in Appendix C for a Maxwellian plasma. The conductivity tensor σ is just the usual textbook result, so the U_L and U_E terms are essentially directly available to any wave propagation code. The U_B terms involve similar, but less familiar, summations over Bessel functions and plasma dispersion functions. In many situations, the U_B terms are small and may be safely neglected. In particular, as noted in Sec II B 1, they vanish for electrostatic modes and are small for modes in the fluid limit.

From the structure of Eqs (32) - (36), it can be seen that F_y can be written in the form

$$F_y = -\frac{\partial}{\partial x}\left(\mathbf{E}^* \cdot A \cdot \mathbf{E}\right) \tag{37}$$

where A is a Hermitian tensor, that can be determined by expanding out the $\mathbf{B}_1$ components in terms of $\mathbf{E}_1$. The tensor A is given in Appendix C.

IV. Application to the IBW

In this section, we make some simple analytical approximations to illustrate a few additional physical points. The cases considered are for an electrostatic IBW and are chosen to illustrate the role of Larmor radius effects, mixed vs. pure polarization and dissipative vs. non-dissipative ions.

A. Mixed polarization case with non-dissipative ions

From Eq. (32) or (37) it is seen that sheared flow drive requires a spatial gradient in the eikonal envelope of the wave, and that U_E and U_B will be dominated by the ion species. A simple limit to consider is that of an electrostatic IBW with non-dissipative ions (i.e. sufficiently far from cyclotron resonance) that acquires its radial gradient from other physics, e.g. electron dissipation.

Under these conditions, χ is Hermitian, and one obtains

$$F_y = \frac{1}{8\pi}\frac{\partial}{\partial x} k_x k_y |\Phi|^2 \left(\frac{\omega}{\Omega_i}\chi_{i\times} - \chi_{i1}\right). \tag{38}$$

where Φ is the electrostatic potential and the components of χ are defined in terms of the polarization $\mathbf{P}$ by $P_x = \chi_1 E_x + i\chi_\times E_y$, $P_y = \chi_2 E_y - i\chi_\times E_x$. The $\chi_{i\times}$ term arises from U_E while χ_{i1} is from U_L. Note that $k_x k_y \neq 0$ is a necessary condition for a non-vanishing poloidal force in the electrostatic limit considered here, i.e. both E_x and E_y must not vanish. Thus "mixed polarization" is essential for a non-vanishing result.

It is instructive to expand Eq. (38) in a power series for small $b \equiv (k_\perp \rho_i)^2$. Defining the shear flow coefficient $S \equiv (\omega/\Omega_i)\,\chi_{i\times} - \chi_{i1}$, one obtains

$$S = -\frac{3b\omega_{pi}^2\Omega_i^2}{(\omega^2-\Omega_i^2)(\omega^2-4\Omega_i^2)}, \qquad b<<1 \tag{39}$$

which vanishes in the fluid limit, $b \to 0$. It is significant that in the fluid limit, the leading order terms of U_E and U_L (each non zero for $b \to 0$) cancel identically, viz., $S(x) \equiv 0$ for all x for which the cold fluid limit applies. In this limit, F_y, V_y and their x derivatives vanish, and there can be no sheared flow drive. Thus, at least in this limit, finite Larmor radius effects are essential to obtain a non-vanishing poloidal force and sheared flow drive. The physical significance of this result is discussed in Sec. V.

Considering the opposite (asymptotic) limit of large b, and further taking only one resonant term in the Bessel function sums, viz. $(\omega - n\Omega_i) << \Omega_i$, one obtains

$$S = -\frac{\omega_{pi}^2}{b\Omega_i^2} + \frac{3\omega\omega_{pi}^2}{2\sqrt{2\pi}\,b^{3/2}\Omega_i^2(\omega-n\Omega_i)} \qquad b>>1. \tag{40}$$

Thus, for $b \sim 1$ and $\omega \sim n\Omega_i$ the typical order of magnitude of sheared-flow drive can be obtained by estimating $S \sim \omega_{pi}^2/\Omega_i(\omega - n\Omega_i)$.

Numerical results for F_y as a function of b are given in Ref. 15, where the present fully kinetic derivation of F_y is also compared with earlier approximate calculations.

B. Dissipative resonant ion case

Next, we consider the limit of an electrostatic IBW with $k_y = 0$, $k_z << k_x$ and dissipative resonant ions, so that the wave electric field is dominantly E_x but finite k_z allows the argument of the plasma dispersion function to be order unity in a narrow layer near a cyclotron harmonic $\omega \approx n\Omega_i$.

For an electrostatic wave we have $U_B = 0$, and inspection of Eq. (33), with $E_y = 0$ and $E_z \approx 0$, shows that U_L also vanishes. Thus the poloidal force is given by the negative gradient of

$$U_E = -\frac{1}{4\Omega}\,|E_x|^2\,\mathrm{Re}\,\sigma_{xx} = |E_x|^2 A_{xx} \tag{41}$$

where σ_{xx} and A_{xx} are given in Appendix C. U_E will maximize when $\mathrm{Im}\,Z(\zeta)$ maximizes, i.e. in the middle of the cyclotron resonance layer where $F_y \propto \partial\,\mathrm{Im}\,Z / \partial x$ vanishes. (Here the argument of the plasma dispersion function is $\zeta = (\omega - n\Omega_i)/\alpha k_z$.) F_y will be of opposite signs on either side of the layer center.

The power dissipation per unit volume is given by

$$P = \langle J_x E_x \rangle = -2\Omega U_E. \tag{42}$$

We can define a “flow drive efficiency” ratio

$$\frac{F_y}{P} \sim \frac{1}{2\Omega L_x} \tag{43}$$

where $L_x = (\partial \ln U_E/\partial x)^{-1} \sim (2 \text{ Im } k_x)^{-1}$ is the dissipation scale length in the cyclotron layer, typically on the order of a few ρ_i. Thus $F_y/P \sim 1/v_i$ for this IBW example.

This flow drive efficiency ratio can be compared to that due to direct absorption of wave momentum, given by the first term in Eq. (14), viz. $F_y/P \sim k_y/\omega$. For a Bernstein wave, which can have phase velocity on the order of v_i, these terms can be comparable if the spectrum has $k_y \sim k_x$. Clearly the direct absorption of wave momentum by Alfvén waves (with phase velocity on the order of the Alfvén velocity) will be much less efficient in a tokamak ($\beta << 1$) than the U_E effect given by Eq. (43) for the IBW. It should also be kept in mind that the *shear* in the flow necessary for turbulent suppression depends on $\partial F_y/\partial x$, giving rise to another factor of L_x in comparing competing mechanisms.

V. Discussion

In the previous section is was shown, for a particular example, that the net poloidal force vanishes for an ideal fluid mode (i.e. a dissipationless fluid mode with $k_\perp\rho \to 0$). To explore the underlying physical reason for the cancellation of the Lorentz (U_L) and pressure tensor (U_E) terms we consider the cold fluid limit for the force given by Eq. (14) with $\Pi = mn\langle\mathbf{uu}\rangle$.

$$F_y = \frac{-1}{16\pi}\frac{\partial}{\partial x}\langle P_x E_y + \Pi_{xy}\rangle = \frac{1}{4}\frac{\partial}{\partial x}\left[nu_x\left(-i\frac{Ze}{\omega}E_y + mu_y\right)^* + cc\right] \tag{44}$$

$$= \frac{\partial}{\partial x}\langle nu_x p_y\rangle$$

where we have employed $P_x = 4\pi i J_x/\omega = 4\pi i Zenu_x/\omega$. In the final form of Eq. (44) we have introduced $\mathbf{p} = m\mathbf{u}\text{-}iZe\mathbf{E}/\omega$ which is the canonical momentum in the radiation gauge. (The momentum $\mathbf{p}$ should not be confused with the polarization vector $\mathbf{P} = \chi\bullet\mathbf{E}$.)

Equation (44) shows that the poloidal (y) force results from the quasilinear averaged radial (x) flux of poloidal momentum. The latter momentum has two contributions, a mechanical piece mu_y and a piece associated with the electromagnetic fields E_y. In order to have a non-vanishing force, it is necessary that u_x and p_y have an in-phase component so that the quasilinear average does not vanish. In cold fluid theory this is not the case, as can immediately be seen by applying Eq. (28) (noting that $\mathbf{a} = Ze\mathbf{E}_1/\omega$) to obtain the cold fluid result

$$p_y = -im\frac{\Omega}{\omega}u_x \tag{45}$$

which implies

$$\langle nu_x p_y \rangle = 0 \tag{46}$$

The addition of finite-Larmor radius effects, viscosity, or dissipation terms to the cold fluid momentum equation generally allows phase shifts to occur which can result in non-zero radial momentum flux and a net poloidal force density. Finite amplitude wave effects, outside the scope of quasilinear theory, that allow orbit stochasticity may also permit a non-zero result.

To return to the IBW case, typically if a long wavelength ($k_\perp\rho_i << 1$) IBW wave is launched into that plasma and allowed to propagate towards an $\omega = n\Omega$ resonance where both dissipation and $k_\perp\rho_i \sim 1$ pertain, we expect $\langle nu_x p_y \rangle$ to be locally large in the resonant cyclotron layer but to vanish outside the layer. As a result, $\int dx\, F_y = 0$ across the layer, and there is no global (radially integrated) poloidal force, but there can be local (in x) forces and local sheared flow generated within the layer. From a fluid mechanical point of view, it is the gyro-viscous and/or dissipative responses of the fluid to the wave-induced stresses that create the back-reaction responsible for the forces acting on the plasma. An inviscid dissipationless fluid creates no such forces. Similar conclusions have recently been reported in the context of a fluid model with $k_\perp^2\rho_i^2$ corrections.[18]

Our general results, Eqs. (33)-(35) combined with Appendix C, include dissipation due to cyclotron and Landau damping, as well as fluid viscosity effects arising from finite $k_\perp\rho_i$.

VI. Conclusions

In this paper, we have presented a systematic formulation of the ponderomotive force on a fluid element due to electromagnetic waves in hot kinetic plasmas describable within the eikonal approximation. Through a series of algebraic manipulations, we have shown how it is possible to construct a general expression for the nonlinear force driving sheared flow, without recourse to an explicit (and very tedious) calculation of the nonlinear distribution function. The main results of our paper are to be found in Eqs. (14) and (26) which give general results in terms of vectors and dyads, and Eqs. (32) - (35) which give the corresponding pseudo-Cartesian result for the poloidal force in the limit $k_y = 0$. The results are obtained in terms of the rf field amplitudes and simple current and pressure moments of the linearized distribution function. We have shown by example in Eq. (39) that it is possible for the force terms and the kinetic pressure terms to cancel in some cases.

The physical reasons for this cancellation have been discussed in Sec. V. Thus, a fully kinetic calculation of the pressure tensor, as obtained in this paper, is required.

As discussed in Sec. I, the present calculations only provide a beginning for a full theoretical understanding of sheared flow drive by rf waves. Many topics, including the important one of calculating rf-induced charge buildup and the resulting induced radial electric fields and poloidal E×B flows, remain for future work.

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Appendix A. Other forms of the ponderomotive force

In the cold fluid limit the rf waves can be described by a simple closure scheme for Π given by

$$\Pi \to \Pi_{fl} \equiv nm\mathbf{uu}, \tag{A1}$$

where $\mathbf{u}$ is the perturbed fluid velocity. In this limit, the PF reduces to an expression involving only $\mathbf{E}$ and $\mathbf{P}$, viz.

$$16\pi\, \mathbf{F} = (\nabla\, \mathbf{E}^*)\cdot\mathbf{P} - \nabla\cdot(\mathbf{PE}^*) - \nabla\cdot\left(\frac{\omega^2}{\omega_p{}^2}\,\mathbf{P}\mathbf{P}^*\right) + \mathrm{cc} \tag{A2}$$

It can also be shown that in the fluid limit the PF can be expressed in several other forms:

$$\mathbf{F} = \langle\mathbf{F}_L\rangle - nm\langle\mathbf{u}\cdot\nabla\mathbf{u}\rangle + m\langle\mathbf{u}\frac{\partial n}{\partial t}\rangle, \tag{A3}$$

$$\mathbf{F} = \langle\mathbf{F}_L\rangle - \nabla\cdot\langle nm\mathbf{uu}\rangle \tag{A4}$$

$$\mathbf{F} = -\, n\nabla\psi + \mathbf{B}\times(\nabla\times\mathbf{M}) \tag{A5}$$

where in the last form[19]

$$\psi = \frac{-1}{2}\, Ze\langle\xi\cdot\mathbf{E}\rangle = \frac{-1}{32\pi n}\mathbf{P}\cdot\mathbf{E}^* + \mathrm{cc} \tag{A6}$$

$$\mathbf{M} = \frac{Zen}{2c} \langle \xi \times \mathbf{u} \rangle = \frac{i\omega\Omega}{32\pi\omega_p{}^2 B}\, \mathbf{P} \times \mathbf{P}^* + cc \qquad \text{(A7)}$$

with $\mathbf{\Omega}$ = ZeB/mc, $\omega_p{}^2 = 4\pi ne^2/m$, $\xi = \mathbf{u}/(-i\omega)$ and $\mathbf{u}$ the solution of

$$-i\omega\mathbf{u} - \mathbf{u} \times \Omega = Ze\mathbf{E}/m, \qquad \text{(A8)}$$

viz. $\mathbf{u} = -i\omega\mathbf{P}/(4\pi Zen)$. It can be shown that

$$-16\pi n\nabla\psi = (\nabla\, \mathbf{E}^*)\cdot\mathbf{P} + cc \qquad \text{(A9)}$$

in fluid theory accounting for the equivalency of the first terms. The proof is fairly straightforward in the spin basis.[20,21] The proof of equivalency of the remaining terms is more involved, but has been shown in Ref. 22.

Yet another form for the PF exists if one considers only the species-summed PF on the whole plasma. This form can be obtained by following the standard electromagnetic derivation for the Maxwell stress tensor.[23] The stress tensor form manipulates $\mathbf{F}_L$ by eliminating nm and $\mathbf{J}$ in terms of the $\mathbf{E}$ and $\mathbf{B}$ fields to obtain

$$4\pi\mathbf{F}_L = \nabla\cdot[\mathbf{EE} + \mathbf{BB} - I(E^2 + B^2)/2 - \partial_t(\mathbf{E} \times \mathbf{B})/c], \qquad \text{(A10)}$$

where here, for notational brevity, $\mathbf{B}$ means $\mathbf{B}_1$. The Poynting term vanishes under a time average to yield

$$16\pi\, \mathbf{F} = \nabla\cdot[\mathbf{EE}^* + \mathbf{BB}^* - I(|E|^2 + |B|^2)/2] + cc - 16\pi\nabla\cdot\langle\Pi\rangle. \qquad \text{(A11)}$$

This form might be useful for conservation laws and global momentum transport issues since it expresses the entire PF as a divergence. While elegant, the stress tensor form is often less useful in practice because it places the burden of various cancellations on the solution of the field equations for $\mathbf{E}$ and $\mathbf{B}$. On the other hand, it makes inter-species cancellations (e.g. due to quasineutrality) explicit.

Appendix B: Diagonal Terms

Although they are not required to obtain the poloidal (sheared flow driving) force, it is physically instructive to examine the diagonal terms of the pressure tensor $\langle\Pi_{avg}\rangle$ defined in Eq. (15) by

$$\langle\Pi_{avg}\rangle = m\int d^3v\, \langle\mathbf{vv}\rangle_\phi \langle f\rangle_{\phi t}, \qquad \text{(B1)}$$

where the subscripts ϕ and t on <> indicate respectively a gyrophase and time average. Here $\langle f\rangle_{\phi t}$ is obtained from the gyrophase averaged Vlasov equation

$$\frac{\partial\langle f\rangle_{\phi t}}{\partial t} + \langle\mathbf{v}\cdot\nabla f\rangle_{\phi t} = -\langle\mathbf{a}_1\cdot\nabla_v f_1\rangle_{\phi t} \qquad \text{(B2)}$$

where we note that the $\Omega\partial/\partial\phi$ term is absent, hence the otherwise small terms involving gradients and time derivatives of f (= f_2) must be retained on the lhs of Eq. (B2). A general solution of this equation is difficult , and fortunately unnecessary, if we are willing to specialize to the case of an eikonal solution for the spatial dependence of the wave fields, with a slowly varying spatial envelope. In this case, as we have already noted, f_2 is driven by the beating of $e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}$ and $e^{-i\mathbf{k}\cdot\mathbf{x}+i\omega t}$ and is therefore free of both rapid space and time variation. Denoting the slow space and time scales by L and T respectively, we further make the assumption v/L << 1/T, which allows the $<\mathbf{v}\cdot\nabla f>_\phi$ term to be dropped. The T scale corresponds to an adiabatic turn-on of the wave fields in the past, and consideration of v/L << 1/T will allow later for a formal correspondence of the results with fluid theory in the relevant limits. We note that since 1/L is formally small, and the ponderomotive force depends on Π through $\nabla\cdot\Pi$, which is already small in one power of 1/L, we neglect all further L dependencies of f_2.

Proceeding with the calculation, in this limit we have

$$\frac{\partial \Pi_{avg}}{\partial t} = -\,m\int d^3v <\mathbf{vv}>_\phi <\nabla_v\cdot\, (\mathbf{a}_1\, f_1)>_t \tag{B3}$$

where the $<>_\phi$ on the last term can be dropped because the gyrophase averaging operation is already implicit in $\int d^3v$. Employing $<\mathbf{vv}>_\phi = I_\perp\, v_\perp^2/2 + \mathbf{bb}\, v_\parallel^2$ and integrating by parts on the ∇_v term, we obtain

$$\frac{\partial \Pi_{avg}}{\partial t} = m\int d^3v <f_1\mathbf{a}_1\cdot\mathbf{v}_\perp I_\perp + 2f_1\mathbf{a}_1\cdot\mathbf{v}_\parallel\, \mathbf{bb}>_t. \tag{B4}$$

This is as far as we can go in general, and gives Π_{avg} (actually its time derivative) in terms of moments of f_1. Taking either the electrostatic ($\mathbf{B}_1 = 0$) or fluid (kv/ω << 1) limit allows $\mathbf{a}_1$ to be pulled outside of the velocity integral and results in

$$\frac{\partial \Pi_{avg}}{\partial t} = mn<\mathbf{a}\cdot\mathbf{u}_\perp I_\perp + 2\mathbf{a}\cdot\mathbf{u}_\parallel\, \mathbf{bb}>_t \tag{B5}$$

where we have dropped the subscript 1 on **a** for notational brevity.

From this point on, we have two goals. One is to show that in the fluid limit, combining the results for Π_{avg} and Π_{osc} one recovers the fluid answer $\Pi^{fluid} = mn<\mathbf{uu}>$. The second is to derive a more general expression for Π_{avg}.

The fluid limit result is obtained most easily by invoking the fluid form for the acceleration given in Eq. (28) of the main text to obtain

$$<\mathbf{a}\cdot\mathbf{u}_\perp> = \frac{\partial}{\partial t}<u_\perp^2/2> \tag{B6}$$

$$\langle \mathbf{a}\cdot\mathbf{u}_{||}\rangle = \frac{\partial}{\partial t}\langle u_{||}^2/2\rangle \tag{B7}$$

so that the equation for Π_{avg} may be integrated to obtain

$$\Pi_{avg}^{fluid} = \frac{mn}{2}\langle u_\perp^2\rangle I_\perp + mn\langle u_{||}^2\rangle \mathbf{bb}. \tag{B8}$$

This indeed combines with Π_{osc}^{fluid} given by Eq. (29) to yield the desired result

$$\Pi^{fluid} = mn\langle \mathbf{uu}\rangle. \tag{B9}$$

More generally, we can work with Π_{avg} kinetically (but here we only consider the electrostatic case where **a** was treated as independent of **v**). In this case we obtain

$$mn\langle \mathbf{a}\cdot\mathbf{u}_\perp\rangle = \langle \mathbf{E}\cdot\mathbf{J}_\perp\rangle \tag{B10}$$

$$mn\langle \mathbf{a}\cdot\mathbf{u}_{||}\rangle = \langle \mathbf{E}\cdot\mathbf{J}_{||}\rangle \tag{B11}$$

$$\frac{\partial \Pi_{avg}}{\partial t} = \langle \mathbf{E}\cdot\mathbf{J}_\perp\rangle I_\perp + 2\langle \mathbf{E}\cdot\mathbf{J}_{||}\rangle \mathbf{bb} + \text{sloshing terms}. \tag{B12}$$

The sloshing terms are obtained from a multiple time scale expansion as follows.

$$\mathbf{J} \rightarrow \mathbf{J} + i\frac{\partial}{\partial t}\frac{\partial \mathbf{J}}{\partial \omega} \tag{B13}$$

where ω describes the fast time scale and ∂/∂t describes the slow time scale, and operates only on the envelope. Thus we obtain

$$\langle \mathbf{E}\cdot\mathbf{J}_\perp\rangle \rightarrow \langle \mathbf{E}\cdot\mathbf{J}_\perp\rangle + \frac{1}{16\pi}\left(\frac{\partial \mathbf{K}}{\partial t}\cdot\mathbf{E}_\perp^* + cc\right). \tag{B14}$$

where

$$\mathbf{K} = \frac{\partial}{\partial \omega}(\omega\chi)\cdot\mathbf{E} \tag{B15}$$

with χ the dielectric tensor susceptibility. To manipulate the sloshing terms further, it is convenient to write $\mathbf{K} = Y\cdot\mathbf{E}$ with Y assumed independent of time. The Hermitian and anti-Hermitian parts of Y are defined by

$$Y^H = \frac{1}{2}(Y + Y^{*t})$$

$$Y^{aH} = \frac{1}{2}(Y - Y^{*t}) \tag{B16}$$

With these definitions we obtain

$$\left(\frac{\partial \mathbf{K}}{\partial t}\cdot\mathbf{E}_\perp{}^* + \mathrm{cc}\right) = \mathrm{Y}^{\mathrm{H}}\,\mathbf{:}\left(\frac{\partial \mathbf{E}}{\partial t}\,\mathbf{E}^* + \mathbf{E}\,\frac{\partial \mathbf{E}^*}{\partial t}\right) +$$

$$\mathrm{Y}^{\mathrm{aH}}\,\mathbf{:}\left(\frac{\partial \mathbf{E}}{\partial t}\,\mathbf{E}^* - \mathbf{E}\,\frac{\partial \mathbf{E}^*}{\partial t}\right), \tag{B17}$$

where the **:** convention is $\mathrm{Y}\mathbf{:BC} = \mathbf{C}\cdot\mathrm{Y}\cdot\mathbf{B}$. The terms involving Y^{aH} are assumed to vanish by construction of the adiabatic turn-on. (Time derivatives in this term correspond to a frequency shift introduced by the slow time scale.) Thus we finally obtain

$$<\mathbf{E}\cdot\mathbf{J}_\perp> \to <\mathbf{E}\cdot\mathbf{J}_\perp> + \frac{1}{32\pi}\frac{\partial}{\partial t}\left(\mathbf{K}^{\mathrm{H}}\cdot\mathbf{E}_\perp{}^* + \mathrm{cc}\right). \tag{B18}$$

A similar procedure is employed to obtain $<\mathbf{E}\cdot\mathbf{J}_{||}>$ and upon substitution results in

$$\frac{\partial \Pi_{\mathrm{avg}}}{\partial t} = <\mathbf{E}\cdot\mathbf{J}_\perp>\mathrm{I}_\perp + 2<\mathbf{E}\cdot\mathbf{J}_{||}>\mathbf{bb} + \mathrm{I}_\perp\frac{1}{32\pi}\frac{\partial}{\partial t}\left(\mathbf{K}^{\mathrm{H}}\cdot\mathbf{E}_\perp{}^* + \mathrm{cc}\right)$$

$$+\, \mathbf{bb}\frac{1}{16\pi}\frac{\partial}{\partial t}\left(\mathbf{K}^{\mathrm{H}}\cdot\mathbf{E}_{||}{}^* + \mathrm{cc}\right). \tag{B19}$$

The presence of the heating terms, viz. $<\mathbf{E}\cdot\mathbf{J}_\perp>$ and $<\mathbf{E}\cdot\mathbf{J}_{||}>$ makes it impossible to time integrate the preceding equation for Π_{avg} explicitly. However, the contributions that these heating terms would make to Π_{avg} would not normally be considered as part of the ponderomotive force. Rather, they represent an irreversible modification of the pressure gradients due to the rf heating. Dropping these terms and denoting the result by Π_{avg}' we obtain

$$\Pi_{\mathrm{avg}}' = \mathrm{I}_\perp\left(\frac{1}{32\pi}\,\mathbf{K}^{\mathrm{H}}\cdot\mathbf{E}_\perp{}^*\right) + \mathbf{bb}\left(\frac{1}{16\pi}\,\mathbf{K}^{\mathrm{H}}\cdot\mathbf{E}_{||}{}^*\right) + \mathrm{cc} \tag{B20}$$

where we recall that

$$\mathbf{K}^{\mathrm{H}} = \frac{\partial}{\partial\omega}(\omega\chi^{\mathrm{H}})\cdot\mathbf{E} \tag{B21}$$

results from the Hermitian contributions to χ.

Finally, it is possible to recover the fluid form of Π_{avg} from the preceding, and this serves as a useful check on the manipulations. We first note that in the fluid limit there is no heating, so that $\Pi_{\mathrm{avg}}' = \Pi_{\mathrm{avg}}$ and χ is automatically Hermitian. The recovery of the fluid limit depends on a remarkable identity for χ^{fluid}, denoted simply as χ in the following.

$$\frac{\partial}{\partial\omega}(\omega\chi) = \frac{\omega^2}{\omega_p{}^2}\,\chi\cdot\chi. \tag{B22}$$

While it is straightforward to show this by explicit substitution of elements of χ in the Cartesian basis, the proof is almost immediate in the spin basis[20] where χ is diagonal with elements

$$\chi_\mu = \frac{-\omega_p{}^2}{\omega(\omega+\sigma_\mu\Omega)} \tag{B23}$$

with σ_μ = -1, 0, or +1 for left circular, parallel and right circular polarizations. Armed with this identity we obtain

$$\mathbf{K}^H\cdot\mathbf{E}_\perp{}^* = \frac{\omega^2}{\omega_p{}^2}(\chi\cdot\mathbf{E}_\perp)\cdot(\chi\cdot\mathbf{E}_\perp{}^*) + cc = 16\pi mn\langle\mathbf{u}_\perp{}^2\rangle \tag{B24}$$

where $\chi\cdot\mathbf{E}_\perp = \mathbf{P}_\perp = 4\pi i\mathbf{J}_\perp/\omega = 4\pi iZen\mathbf{u}_\perp/\omega$ has been employed. A similar procedure for the parallel components is used to obtain the final result

$$\Pi_{avg}{}'^{\,fluid} = \Pi_{avg}{}^{fluid} = \frac{mn}{2}\langle u_\perp{}^2\rangle I_\perp + mn\langle u_{||}{}^2\rangle\mathbf{bb}, \tag{B25}$$

as before.

Appendix C: Explicit Results for the σ, p and A Tensors

For a Maxwellian plasma, the linearized Vlasov equation may be solved by standard techniques to obtain the first order distribution function

$$f_1 = \frac{iZef_0}{T}\sum_{nm}\left(\frac{n\Omega}{k_x}J_nE_x + iv_\perp J_n{}'E_y + v_zJ_nE_z\right)\frac{J_m e^{i(m-n)\phi}}{\omega-k_zv_z-n\Omega} \tag{C1}$$

where $J_n = J_n(a)$ is a Bessel function with $a = k_\perp v_\perp/\Omega$ and we have specialized to the case $k_y = 0$. From Eq. (C1) the relevant velocity moments can be taken to obtain the current $\mathbf{J} = \sigma\mathbf{E}$ and the pressure p_{ij} defined in Eq. (31). It is useful to carry out the gyrophase integrals first, and to note that the velocity integrations are separable in $v_\perp$ and $v_{||}$. After some straightforward but tedious algebra, one obtains for the elements of σ:

$$\sigma_{xx} = -i\frac{\omega_p{}^2}{4\pi}\sum_n \frac{n^2}{b}\Lambda_n\frac{Z}{k_z\alpha} \tag{C2a}$$

$$\sigma_{xy} = \frac{\omega_p{}^2}{4\pi}\sum_n n\Lambda_n{}'\frac{Z}{k_z\alpha} \tag{C2b}$$

$$\sigma_{xz} = i \frac{\omega_p^2}{4\pi} \sum_n \frac{n\Omega}{k_x v_i^2} \Lambda_n \frac{Z'}{2k_z} \tag{C2c}$$

$$\sigma_{yx} = -\sigma_{xy} \tag{C2d}$$

$$\sigma_{yy} = -i \frac{\omega_p^2}{4\pi} \sum_n \left(\frac{n^2}{b} \Lambda_n - 2b\Lambda_n' \right) \frac{Z}{k_z \alpha} \tag{C2e}$$

$$\sigma_{yz} = \frac{\omega_p^2}{4\pi} \sum_n \frac{k_x}{\Omega} \Lambda_n' \frac{Z'}{2k_z} \tag{C2f}$$

$$\sigma_{zx} = \sigma_{xz} \tag{C2g}$$

$$\sigma_{zy} = -\sigma_{yz} \tag{C2h}$$

$$\sigma_{zz} = i \frac{\omega_p^2}{4\pi} \sum_n \Lambda_n \frac{\zeta Z'}{k_z \alpha} \tag{C2i}$$

Here $\Lambda_n = \Lambda_n(b) = I_n(b) \exp(-b)$, $Z = Z(\zeta)$ is the plasma dispersion function, $\zeta = (\omega - n\Omega)/k_z\alpha$, $\alpha = \sqrt{2}\, v_i$, $v_i^2 = T/m$ and $b = k_\perp^2 v_i^2/\Omega^2$.

For the linearized pressure tensor we find

$$p_{xyx} = -\frac{Zen_0}{T} \sum_n \frac{n^2\Omega^3}{k_x^3} \frac{Z}{k_z\alpha} (b\Lambda_n' - \Lambda_n) \tag{C3a}$$

$$p_{xyy} = -i \frac{Zen_0}{T} \sum_n \frac{n\Omega^3}{k_x^3} \frac{Z}{k_z\alpha} (-b\Lambda_n' + n^2\Lambda_n - 2b^2\Lambda_n') \tag{C3b}$$

$$p_{xyz} = \frac{Zen_0}{T} \sum_n \frac{n\Omega^2}{k_x^2} \frac{Z'}{2k_z} (b\Lambda_n' - \Lambda_n) \tag{C3c}$$

$$p_{yzx} = \frac{Zen_0}{T} \sum_n \frac{n\Omega^2}{k_x^2} \frac{Z'}{2k_z} b\Lambda_n' \tag{C3d}$$

$$p_{yzy} = i\,\frac{Zen_0}{T}\sum_n v_i{}^2\,\frac{Z'}{2k_z}\left(\frac{n^2}{b}\Lambda_n - 2b\Lambda_n{}'\right) \tag{C3e}$$

$$p_{yzz} = \frac{Zen_0}{T}\sum_n \frac{\Omega}{k_x}\,\frac{\alpha\zeta Z'}{2k_z}\,b\Lambda_n{}' \tag{C3f}$$

$$p_{xzx} = i\,\frac{Zen_0}{T}\sum_n \frac{n^2\Omega^2}{k_x{}^2}\,\frac{Z'}{2k_z}\,\Lambda_n \tag{C3g}$$

$$p_{xzy} = -\,\frac{Zen_0}{T}\sum_n \frac{n\Omega^2}{k_x{}^2}\,\frac{Z'}{2k_z}\,b\Lambda_n{}' \tag{C3h}$$

$$p_{xzz} = i\,\frac{Zen_0}{T}\sum_n \frac{n\Omega}{k_x}\,\frac{\alpha\zeta Z'}{2k_z}\,\Lambda_n \tag{C3i}$$

Combining the results of Eqs. (C2) and (C3) with Eqs. (32) – (36) we obtain an explicit result for the tensor A defined by Eq. (37). It is convenient to keep the contributions from U_L and U_E separate from U_B since the latter do not contribute for purely electrostatic waves.

$$A \equiv A_{LE} + A_B \tag{C4}$$

$$A_{LE} = \frac{\omega_p^2}{16\pi b\alpha\Omega k_z}\begin{pmatrix} -\operatorname{Im} f_{21} & \tau f_{21}^{*} - b\operatorname{Re} f_{11}' & -\frac{i}{4}\sqrt{2b}\, f_{12} \\ CT & \operatorname{Im}(f_{21} - 2b^2 f_{01}') - 2b\tau\operatorname{Im} f_{11}' & -\tau\sqrt{b/2}\, f_{12} + \frac{b}{4}\sqrt{2b}\, f_{02}' \\ CT & CT & 0 \end{pmatrix} \tag{C5}$$

$$A_B = \frac{\omega_p^2}{16\pi b\alpha\Omega k_z}\begin{pmatrix} \frac{1}{2}\xi\tau\,\mathrm{Im}\,f_{22} & -\tau f_{21}^{*} + \frac{1}{2}b\xi\tau\,\mathrm{Re}\,f'_{12} + b\tau f'^{*}_{21} & -\frac{i}{4}\sqrt{2b}\,\tau(\xi f_{13} + f_{22}^{*}) \\ CT & -\frac{1}{2}\xi\tau\,\mathrm{Im}\,f_{22} - 2\tau\,\mathrm{Im}\,f_{31} + b^2\xi\tau\,\mathrm{Im}\,f'_{02} + 2b(1+2b)\tau\,\mathrm{Im}\,f'_{11} & \tau\sqrt{b/2}\,f_{12} + \frac{b\tau}{4}\sqrt{2b}\,\xi f'_{03} - b\tau\sqrt{b/2}\,(f'_{12} + f'^{*}_{12}/2) \\ CT & CT & -b\tau\,\mathrm{Im}\,f_{13} \end{pmatrix} \quad \text{(C6)}$$

where we have defined some additional symbols for notational brevity:

$$\tau = \Omega/\omega \quad \text{(C7)}$$

$$\xi = \alpha k_z/\Omega.$$

The functions f_{pq} are combinations of Bessel and Z functions given by

$$f_{pq} = \sum_n n^p \Lambda_n(b) \begin{cases} Z(\zeta) & q = 1 \\ Z'(\zeta) & q = 2 \\ \zeta Z'(\zeta) & q = 3 \end{cases} \quad \text{(C8a)}$$

$$f'_{pq} = \frac{\partial}{\partial b} f_{pq} \quad \text{(C8b)}$$

The matrix elements denoted CT in Eqs. (C5) and (C6) can be obtained by the complex transpose (Hermitian) rule.

From the structure of A we can make several general observations. Since the diagonal elements of A are proportional to the imaginary part of the Z-functions, it follows that dissipation is required for waves which are linearly polarized, i.e. having the electric field purely in the x or y directions. (Recall that $k_y = 0$ is assumed throughout this Appendix; therefore, when k_z is small, x and y polarizations correspond to electrostatic and electromagnetic waves respectively.) With purely z polarization, only electromagnetic waves can produce a poloidal force because the zz component of A_{LE} vanishes. Although not immediate from inspection, it is easy to show that dissipation is also required for waves that have pure circular polarization. However, an examination of the off-diagonal elements shows that in general the mixed polarization case can produce a poloidal force when dissipation is absent, provided one does not take the limit b = 0. This result was already illustrated in Sec. IV for a model IBW problem. Finally, in the electromagnetic case where b, ζ, the Bessel functions, and the Z functions are regarded as order unity, the matrix elements are all comparable and thus there is no *a priori* optimal polarization. Results will depend on the particular mode in question. A comparative analysis of the sheared-flow-drive properties of propagating eigenmodes is beyond the scope of this paper.

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