Ion temperature gradient mode test of the 2DX code

D. A. Baver and J. R. Myra

Lodestar Research Corporation, Boulder Colorado 80301

Maxim Umansky

Lawrence Livermore National Laboratory

1 Introduction

This test was devised to verify the ability of the 2DX eigenvalue code to correctly solve a simple fluid model relevant to edge turbulence in tokamaks. Since the functionality of the 2DX code depends on both the source code itself and the input file defining the system of equations to solve (structure file), this test demonstrates both. Since a similar test was performed on an earlier version of 2DX, this verifies that the current version retains this functionality. Moreover, since the structure file for this test represents a subset of a more general 6-field model, many of the terms in that test are also verified.

This test compares 2DX results to BOUT simulations, and to exact analytic results based on a simplified eigenvalue problem.

2 Description

2.1 Code structure

The 2DX code is a highly flexible eigenvalue solver designed for problems relevant to edge physics in toroidal plasma devices. Its flexibility stems from the use of a specialized input file containing instructions on how to set up a particular set of equations. Because of this, the 2DX code permits model equations to be changed without altering its source code. The drawback to this approach is that any change to the structure file represents a potential source of error, necessitating re-verification. This problem is offset by the fact that the source code remains unchanged, thus testing one structure file builds confidence in the underlying code that interprets the structure file. Also, structure files can be translated into analytic form, thus allowing the user to verify that the file contains the equations intended.

The structure file contains two main parts: an elements section, which constructs the differential operators and other functions used in a particular set of equations, and a formula section, which assembles these into an actual set of equations. This separation means that elements can be recycled in other structure files. By testing one structure file, one builds confidence in the elements used in that file. The main source of error when switching to a different structure file then is in the formula section, which can be manually verified by translating into analytic form.

Regardless of the content of the structure file, the 2DX code is fundamentally a finite-difference eigenvalue solver. As such, it is subject to the limitations of any code of its type.

2.2 Model equations

For this test we use the following model equations [1]-[2]:

$$\gamma \nabla_\perp^2 \delta\Phi = -\frac{B^2}{n} \partial_\parallel \nabla_\perp^2 \delta A \tag{1}$$

$$\gamma \delta n = -n \partial_\parallel \delta u - \partial_\parallel \nabla_\perp^2 \delta A \tag{2}$$

$$\gamma \delta u = -\frac{1}{n} \nabla_\parallel \delta p \tag{3}$$

$$\gamma \delta T_i = -\delta v_E \times \nabla T_i - \frac{2}{3} T_i \partial_\parallel \delta u \tag{4}$$

$$-\gamma \nabla_\perp^2 \delta A = -\mu n \nabla_\parallel \delta\Phi + \mu T_e \nabla_\parallel \delta n \tag{5}$$

where

$$\delta p = (T_e + T_i)\delta n + n(\delta T_e + \delta T_i) \tag{6}$$

$$C_r = \mathbf{b} \times \kappa \cdot \nabla = -\kappa_g R B_p \partial_x + i(\kappa_n k_b - \kappa_g k_\psi) \tag{7}$$

$$\nabla_\perp^2 = -k_b^2 - \jmath B(k_\psi - i\partial_x R B_p)(1/\jmath B)(k_\psi - i R B_p \partial_x) \tag{8}$$

$$\partial_\parallel Q = B \nabla_\parallel (Q/B) \tag{9}$$

$$\nabla_\parallel = \jmath \partial_y \tag{10}$$

$$\delta v_E \cdot \nabla Q = -i \frac{k_z (R B_p \partial_x Q)}{B} \delta\Phi \tag{11}$$

$$\nu_e = .51 \nu_r n / T_e^{3/2} \tag{12}$$

2.3 Boundary conditions

This test case uses phase-shift periodic boundary conditions in the parallel direction, and zero-derivative boundary conditions in the radial direction. The phase shift in the parallel direction is given by:

$$e^{i2\pi nq} \tag{13}$$

2.4 Profile setup

The formulas in Eq. 1-5 are normalized to Bohm units. Values are converted by dividing input distances by ρ_s, and input magnetic fields are in Tesla. Output eigenvalues are multiplied by ω_{ci}. Resistivity is given by the formula:

$$\nu_r = \frac{\mu_u}{.51\sigma} \tag{14}$$

where

$$\sigma = 1.96\frac{\omega_{ce}}{\nu_{ei}} \tag{15}$$

The geometry used is a periodic slab. Curvature effects are included in the equation set, but curvature is set to zero. Zero-derviative boundary conditions are used in the radial direction, and the domain is set to only two grid cells wide in that direction. This is done so as to approximate a 1-D problem using a 2-D code, and because the 2DX code cannot simulate domains that are only one grid cell wide in either direction.

3 Analytic results

Since the ITG model is tested in a homogenous domain, it can be solved analytically by taking a Fourier transform in both directions. This allows the system of differential equations to be reduced to algebraic matrix equations. Assuming that $\nabla_\perp^2 = -k_b^2$, this yields an eigenvalue problem of the form $Ax = \gamma x$, where A is:

$$\begin{bmatrix} 0 & -ink_y & 0 & 0 & ik_y \\ -ik_y(T_e+T_i)/n & 0 & -ik_y & 0 & 0 \\ ik_bT_i'/n & -(2/3)iT_ik_y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -ik_y/nk_b^2 \\ i\mu nk_y & 0 & 0 & -i\mu nk_y & 0 \end{bmatrix} \tag{16}$$

and x is $[\delta\Phi, \delta n, \delta u, \delta T_i, \delta A]$.

These can be solved using standard eigenvalue solving routines. The results of this calculation are shown in Figs. 1 along with the numerical results from 2DX.

4 Numerical results

The code was tested by running one case and sorting the eigenvalues by parallel wavenumber. This allowed for a plot of growth rate as a function of wavenumber using only a single run of the code. The parameters used in this run are as follows:

$n = 1$
$T_e = 1$
$T_i = 1$
$T_i' = -1$
$B = 1$
$RB_p = 1$
$k_b = 1$
$\jmath = .07$
$\omega_{ci} = 1$

The profiles used in this test case are also shown in table 1.

The results are shown in Fig. 1, and the raw data for this run is shown in table 2. The vertical axis here is normalized growth rate, whereas the horizontal axis is normalized parallel wavenumber. For analytic and BOUT results, the parallel wavenumber is specified in order to calculate each specific data point. For the 2DX results, parallel wavenumber is not specified, and the eigenvalue solver returns a large number (in this case 60) of relevant eigenmodes. From the eigenvector of each eigenmode one can calculate a parallel wavenumber, and that value is used to determine the horizontal position of each data point. In this graph, there are fewer than 60 unstable modes in the system, so a number of modes on the neutral branch of the dispersion relation are returned as well.

References

[1] B. Coppi, M.N. Rosenbluth and R.Z. Sagdeev, Phys. Fluids **10**, 582 (1967).

[2] S. Hamaguchi and W. Horton, Phys. Fluids B **2**, 1833 (1990).

nx	4	ny	16
dx	.693242	dy	.392699
ω_{ci}	4.79×10^7	m	.5
nx_{LCS}	4	nx_{mis}	0
j_1	1	j_2	16
Γ	0	μ_{ii}	0
μ	2	ν_r	.00131267
δ^2_{er}	1	S_E	1
Λ_1	1	Λ_2	0
q	1.5	k_b	-.144249

Table 1: Profile functions and parameters used in the ITG test case.

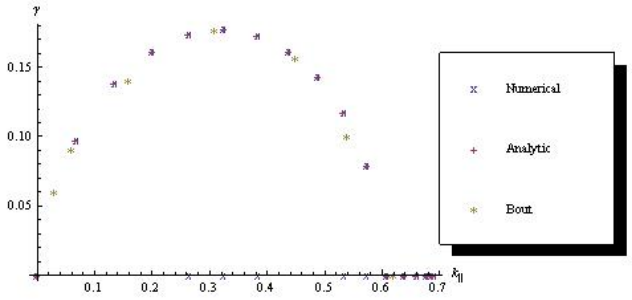

Figure 1: Growth rate vs. k_y for the ITG model

k_z	γ	k_z	γ	k_z	γ
0.325609	0.177017	0.325609	0.177017	0.325609	0.177017
0.325609	0.177017	0.264332	0.173671	0.264332	0.173671
0.264332	0.173671	0.264332	0.173671	0.38375	0.172702
0.38375	0.172702	0.38375	0.172702	0.38375	0.172702
0.200509	0.161416	0.200509	0.161416	0.200509	0.161416
0.200509	0.161416	0.438196	0.161362	0.438196	0.161362
0.438196	0.161362	0.438196	0.161362	0.488422	0.143084
0.488422	0.143084	0.488422	0.143084	0.488422	0.143084
0.134755	0.137856	0.134755	0.137856	0.134755	0.137856
0.134755	0.137856	0.533943	0.117025	0.533943	0.117025
0.533943	0.117025	0.533943	0.117025	0.0677036	0.0973153
0.0677036	0.0973153	0.0677036	0.0973153	0.0677036	0.0973153
0.574323	0.0793186	0.574323	0.0793186	0.574323	0.0793186
0.574323	0.0793186	0	0	0	0
0.574323	0	0.574323	0	0.690732	0
0.574323	0	0.533943	0	0.638154	0
0.67746	0	0.609172	0	0.66099	0
0.533943	0	0.533943	0	0.325609	0
0.264332	0	0.38375	0	0.687406	0
0.67746	0	0.325609	0	0.66099	0

Table 2: Growth rate vs. k_y for the ITG model